AD-A014 151

DESIGN TECHNIQUE FOR IMPROVED BANDWIDTH MOVING TARGET INDICATOR PROCESSORS IN SURFACE RADARS

Donald W. Burlage, et al

Army Missile Research, Development and Engineering Laboratory
Redstone Arsenal, Alabama

11 June 1975

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE

252076

ADA014151

TECHNICAL REPORT RE-75-35

DESIGN TECHNIQUES FOR IMPROVED BANDWIDTH MOVING TARGET INDICATOR PROCESSORS IN SURFACE RADARS

Donald W. Burlage and Ronald C. Houts
Advanced Sensors Directorate
US Army Missile Research, Development and Engineering Laboratory
US Army Missile Command
Redstone Arsenal, Alabama 35809

20 June 1975

Approved for public release; distribution unlimited.

U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER RE-75-35	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DESIGN TECHNIQUE FOR IMPROVED BANDWIDTH MOVING TARGET INDICATOR PROCESSORS IN SURFACE RADARS		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER RE-75-35
7. AUTHOR(s) Donald W. Burlage and Ronald C. Houts		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Commander US Army Missile Command ATTN: AMSMI-RPR Redstone Arsenal, Alabama 35809		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA 1M262303A214 AMCMS 632303.11.21401
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 11 June 1975 13. NUMBER OF PAGES 20
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Moving target indicator
Finite-impulse response filter
Chebyshev design
Improvement factor
Clutter
Digital filter
Transversal filter
MTI radar
Doppler filter

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A technique is presented for assigning the number and values of a set of transversal digital filter weights used with a moving target indicator filter to remove clutter. The approach involves a tradeoff between the amount of clutter rejection and useable bandwidth over which a moving target indicator filter satisfactorily passes a doppler signal; i.e., a specified detection probability is met or exceeded. The useable bandwidth is enhanced by an integrator which sums the B outputs from an N-tap, fixed window moving target indicator filter during a beam dwell of NB pulses. The procedure is

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block 20 Abstract (Concluded)

applicable to any moving target indicator radar which employs a transversal moving target indicator filter and a reasonable number of returns/beam dwell. The narrow-band clutter example (Gaussian power spectrum, $\sigma = 5$ Hz) indicates an improvement in useable bandwidth from the 62% achievable with 16 outputs from a conventional three-pulse canceller to 80.8% for five outputs from a nine-tap Chebyshev filter, while only reducing the clutter rejection from 92 to 69 dB. Even larger bandwidth utilizations are possible for the Chebyshev moving target indicator designs when larger passband ripple can be tolerated. Similar results hold for the wideband clutter design ($\sigma = 100$ Hz); however, even and odd numbers of taps must be considered and variable stopband weighting can be utilized effectively. Listings for computer programs used in the design and analysis are included.

DISPOSITION INSTRUCTIONS

DESTROY THIS REPORT WHEN IT IS NO LONGER NEEDED. DO NOT RETURN IT TO THE ORIGINATOR.

DISCLAIMER

THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

TRADE NAMES

USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL INDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.

CONTENTS

1. Moving Target Indicator Signal Processor

The function of a moving target indicator (MTI) signal processor is to extract a doppler frequency signal from a much stronger low-frequency noise, hereafter referred to as clutter. The conventional performance measure of an MTI processor is the improvement factor (I); however, it will be shown that this is not a complete measure of performance. The section concludes with a description of the processor.

a. Clutter Characterization

The clutter emanates from a variety of sources; however, for purposes of discussion only two categories will be considered. Narrowband clutter consists of a DC spectral component plus an AC spectrum which is only a few hertz wide. The AC spectrum is typically presumed to be Gaussian with zero mean and a standard deviation (σ) which is 10 Hz or less. This clutter spectrum is generally the result of ground returns and the ratio of DC-to-AC clutter power is defined as m^2. The second category, wideband clutter, is also assumed to have a Gaussian power-density spectrum; however, σ is more typically in the range of 50 to 150 Hz and there is no DC component. Wideband clutter is typically the result of natural turbulence such as rain storms or man-made interference such as chaff.

b. Performance Measure

The performance of an MTI signal processor is typically measured in terms of an improvement factor (I) which is defined as the output ratio of signal-to-clutter power (SCR)in dB minus the corresponding ratio at the processor input. The input SCR is typically quite small, e.g., -30 dB. To remove the clutter, the MTI processor must act as a high-pass filter (HPF) with a stopband extending from DC to ~3σ Hz. If the filter were ideal, it would remove the DC component and 99.75% of the AC clutter power without affecting the doppler signal, provided its spectrum was above 3σ Hz. For example, if the clutter return was narrowband resulting from rocky terrain ($m^2 = 30$) [1] the ideal MTI would provide an improvement (I = 40 dB) and the output SCR = 10 dB. Additional improvement could be obtained by extending the stopband edge past 3σ Hz; however, this would diminish the range over which a doppler signal could be detected. Hence, even with ideal filtering, a trade-off exists between clutter rejection and signal detection. This trade-off is not evident in the conventional definition of I, because the signal gain is typically calculated as an average over the entire frequency band (DC to PRF), where PRF is also denoted as (1/T) Hz, T being the time between two pulses. This averaging philosophy is not realistic in the sense that the return from a target is typically assumed to be a pure tone or at most a narrowband signal. However, it is understandable in that the doppler frequency is proportional to the cosine of the angle between the radar beam and the

Preceding page blank

target velocity vector, hence, can vary from DC to an upper limit which depends upon the target's velocity vector. Frequency-response presentations are often normalized with respect to the PRF and extend only to 0.5 since the digital filter which approximates the HPF is symmetric about PRF/2. The PRF used in this report is 5 kHz which is used by MICOM'S experimental array radar (EAR) [2], and all responses are presented from DC to either 2.5 or 5 kHz.

The ability of the radar to detect the doppler return is measured in terms of the detection probability (P_d) which is a function of the signal-to-noise ratio out of the MTI processor and the false-alarm probability (P_f). The background noise is assumed to be broadband with uniform power density. The threshold is adjusted to provide a fixed P_f in the presence of this noise. Thus, if any narrowband clutter were to pass through the MTI filter it would violate the broadband noise premise. Consequently, in the analysis which follows, it is assumed that the improvement factor specification (I) is sufficient to suppress the clutter spectrum below the broadband noise which is transferred from the radar receiver to the detector. It is further assumed that I is specified in terms of the AC clutter spectra, i.e., $m^2 = 0$. Any DC component which might exist is presumed to be completely suppressed by the MTI filter.

c. Processor Model

To combat a lack of phase coherence at the IF frequency, the conventional MTI signal processor has both in-phase (I) and quadrature (Q) channels, each complete with identical analog-to-digital converters and digital MTI filters. The outputs are then recombined using a circuit which closely approximates the function $(I^2 + Q^2)^{1/2}$. The loss due to a lack of phase coherence is typically one decibel. In the analysis which follows, the MTI digital filter will be designed and its improvement factor calculated without regard to the I and Q channel circuitry and the one-decibel loss factor will be included in system losses. The program IMPFACT, which is described in Appendix A, is designed to compute the improvement factor for a particular N-tap MTI filter with given weights $\{h_i\}$, $i = 1,2,\ldots,N$ and a specified PRF. The user can also select Gaussian or uniform power-density spectra clutter models and must specify the assumed standard deviation (σ_d) plus the range of σ values over which a sensitivity study (I versus σ) is to be performed. The processor typically includes an integrator which sums R outputs from the MTI filter. If each output is calculated using the same returns as the previous output except for one new return which replaces the oldest pulse, then the MTI is classified as "moving window." For the analysis which follows, it will be assumed that each output is computed from a new set of N successive pulse returns, in which case the MTI filter is termed "fixed window." Since the number of pulses transmitted in a fixed beam position is limited, increasing the filter size N decreases the available number of outputs, thereby reducing the integration gain (IG) which is

$$IG = 10 \log(R) \quad . \tag{1}$$

2. MTI Filter Design Techniques

Three basic approaches to designing the MTI filter are considered. The three-pulse canceller works on the intuitive approach that by weighting three successive returns such that the response is zero at DC and maximum at PRF/2, the clutter component will cancel, whereas the doppler signal will be relatively unaffected. The covariance technique assumes a Gaussian clutter spectrum from which a covariance matrix can be formed. The filter weights which maximize I can be extracted from this matrix. The Chebyshev design method uses a minimax frequency error approximation to the ideal HPF described previously. The amplitude responses, H(f), for the three designs are compared in Figure 1 and the corresponding sensitivity studies (I versus σ) are shown in Figure 2. These figures clearly demonstrate the trade-off between I and useable doppler bandwidth.

a. Three-Pulse Canceller

The conventional approach to MTI filtering has been the three-pulse canceller (TPC) with weights $\{h_i\} = \{-1,+2,-1\}$. Intuitively, it is based on the premise that the clutter samples are essentially constant from pulse to pulse, whereas the target's doppler signal is oscillating with a frequency (f_d) which causes the pulse samples to vary in amplitude. Barton [3] has analyzed H(f) and I for this canceller. Although with narrowband clutter it is possible to achieve I > 90 dB, the signal gain can easily be less than unity. This gain is not to be confused with the average signal gain $\sum h_i^2 = 6$. The signal is attenuated whenever the frequency response, $H(f_d) = K \sin^2(\pi f_d T) < 1$, where K = 4; therefore, a signal gain results for the TPC only when $0.167 < f_d T < 0.833$. Furthermore, since any background noise (assumed to be broadband with low power relative to clutter) is also amplified by the frequency response of the canceller, it is conventional to scale all coefficients by $\sqrt{\sum h_i^2}$, which insures zero decibel noise gain. Under these conditions K = 1.633 for the normalized TPC and a signal gain only occurs when $0.286 < f_d T < 0.714$; i.e., a loss is experienced over approximately 55% of the PRF range, whereas the narrowband clutter might occupy only 1% of the band. Evidently, the TPC is rather inefficient in spite of an impressive I value. The problem

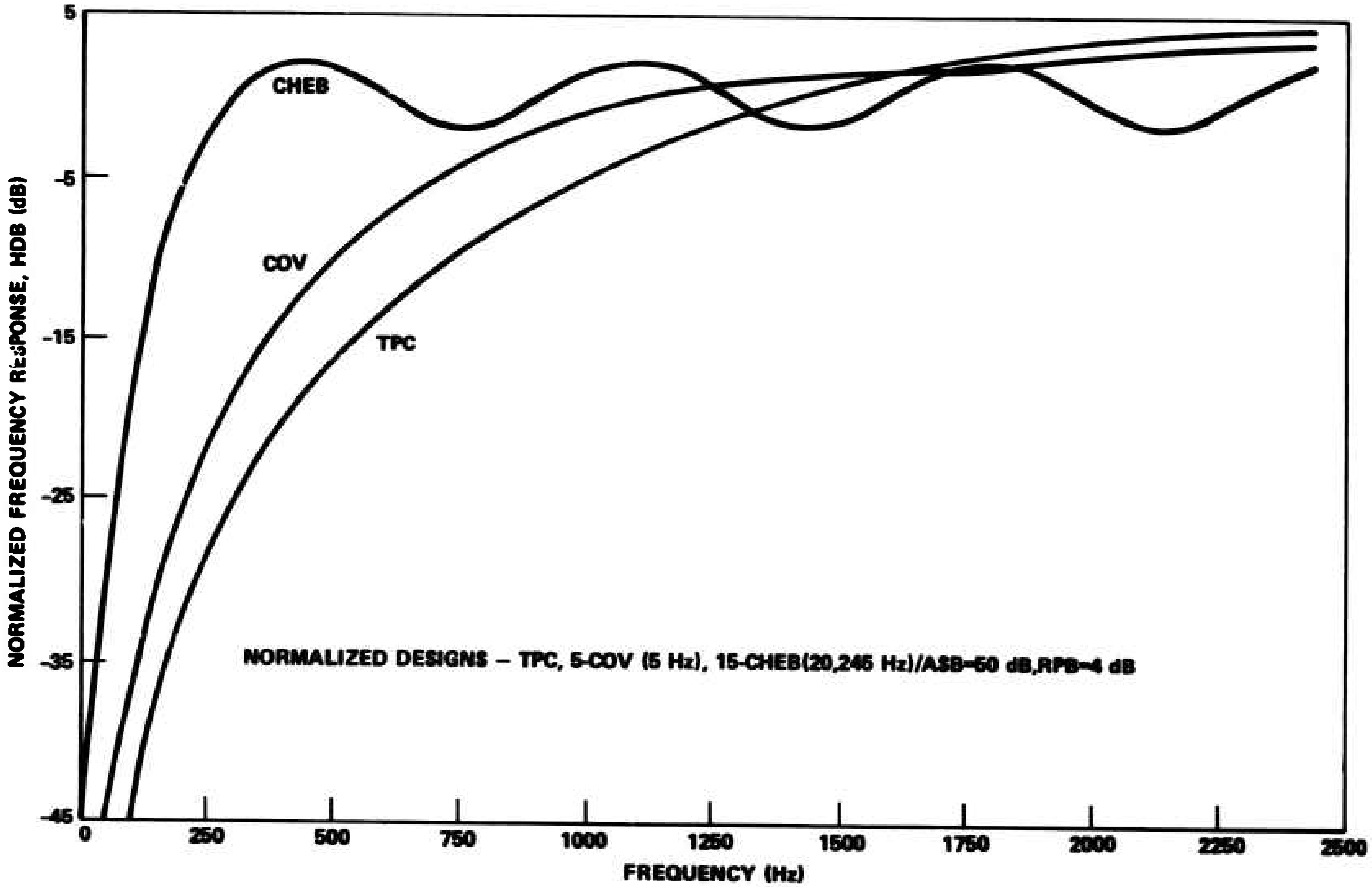

Figure 1. Comparison of frequency responses for three narrowband-clutter filter designs.

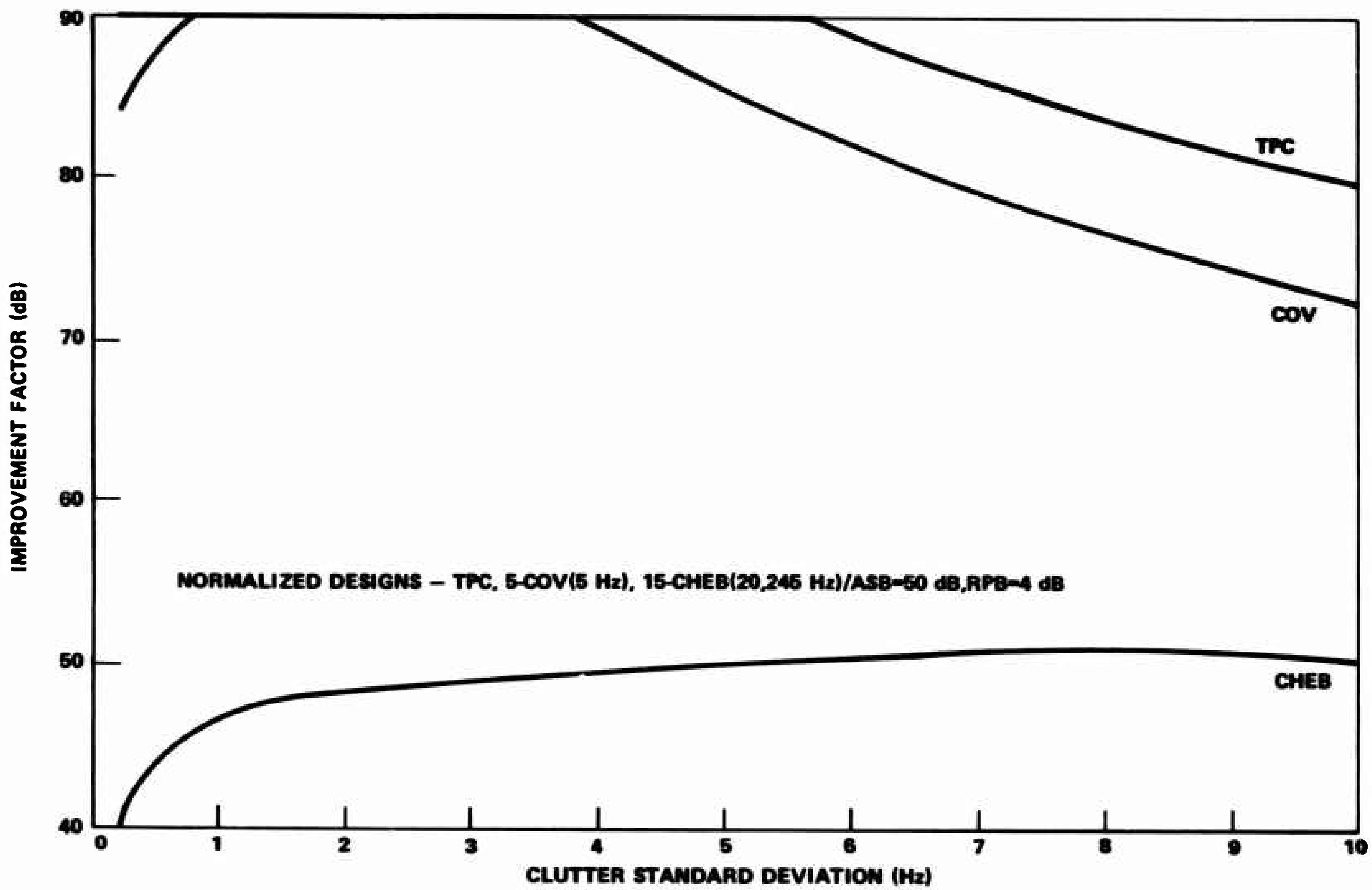

Figure 2. Sensitivity studies for three narrowband-clutter designs.

stems from the fact that the bulk of I is generated by clutter rejection and is not significantly altered by the average signal gain, which is typically less than one decibel. Also, H(f) is independent of the clutter spectrum and I is drastically reduced for wideband clutter. For example, I = 91 dB at σ = 5 Hz, but only 39 dB at σ = 100 Hz.

b. Covariance Design

An improved MTI filter using N weights is described by Capon [4]. The weights for this filter are selected according to an algorithm which maximizes I by minimizing the clutter power. The weights are normalized such that the background-noise power gain is unity or zero decibel. The clutter power-density spectrum is assumed to be Gaussian; consequently, the covariance function is

$$\rho(\tau) = \frac{1}{C} \exp[-(2\pi\sigma\tau)^2/2] \quad , \qquad (2)$$

where C is the input clutter power. For any N-tap filter the corresponding N × N clutter covariance matrix can be generated with τ restricted to be kT, k = 0, 1,...,N-1, and the desired weights are the elements of the eigenvector corresponding to the smallest eigenvalue of the covariance matrix. Although this design is a decided improvement over the TPC concept, Capon points out that the resulting passband frequency response may be poor.

Amplitude responses for several covariance designs are shown in Figure 3. The filters were designed either for narrowband clutter (σ_d = 5 Hz, N = 3,5) or wideband clutter (σ_d = 100 Hz, N = 6,15), and the response for N = 3 is equivalent to that of the normalized TPC. This figure demonstrates that the stopband is dependent on σ_d, and that increasing N provides some improvement in the passband. Interestingly, attempts to increase the number of weights for σ_d = 5 Hz resulted in designs which degenerated to four or five nonzero values. Although designs were obtained for N > 5 with σ_d = 100 Hz, the results were unpredictable in that increasing N beyond six did not necessarily imply better I or a flatter H(f) in the passband. Data indicating the improvement for various σ, I(σ), and selected values of the frequency response in decibel, HBD(f), for a wideband clutter design with N = 3,4,...,9 are listed in Table 1. A strong correlation exists between I(σ) and HDB(500); however, HDB(500) did not correlate with increasing N. More predictable results were obtained when the clutter σ_d was increased to 300 Hz, or equivalently the PRF reduced by a factor of three.

The covariance-filter design algorithm is described in Appendix B. From the user's viewpoint, it is quite straightforward requiring as input values of N, σ_d, and the PRF. Output includes the normalized

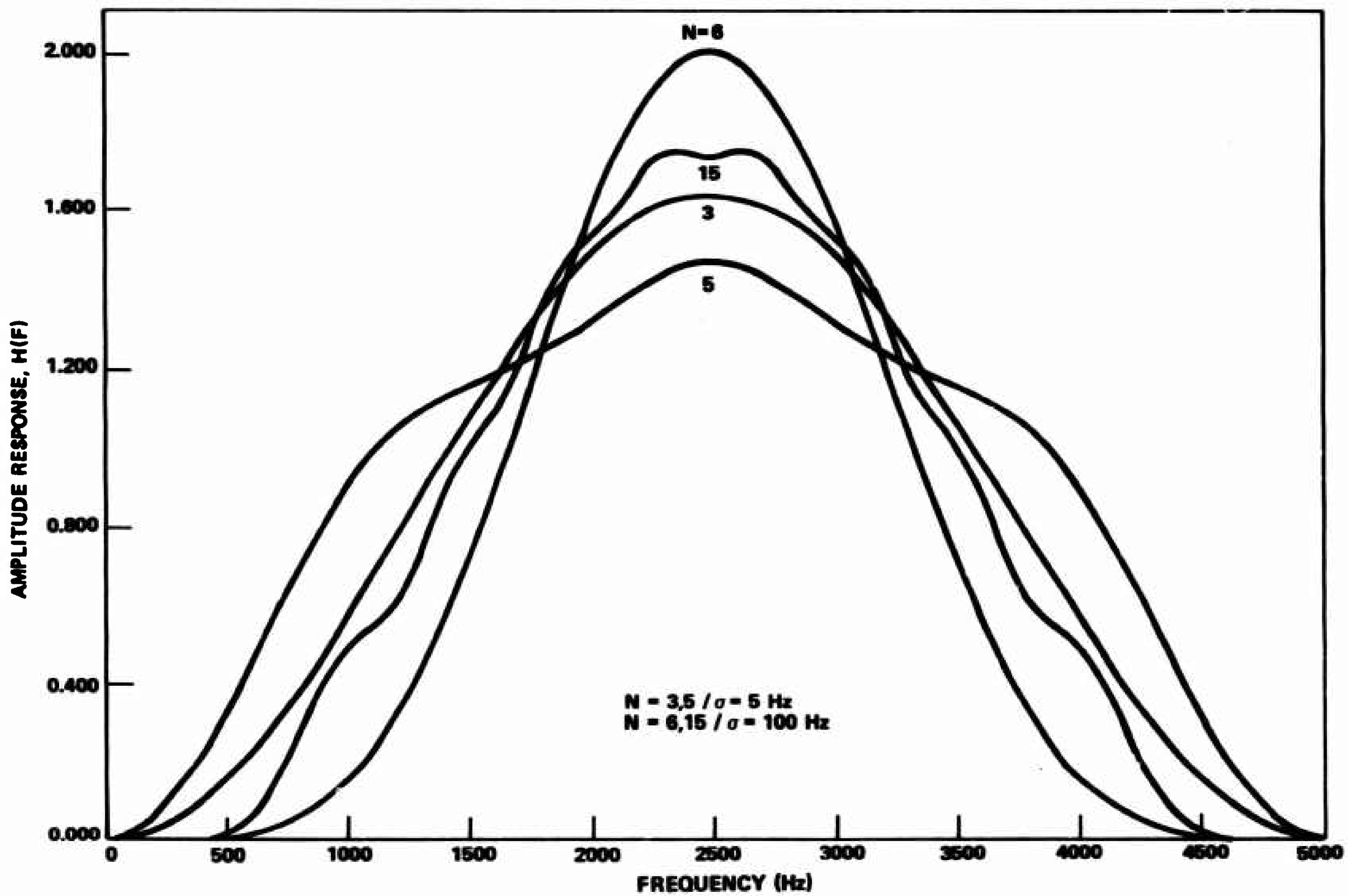

Figure 3. Amplitude responses for N-tap covariance filters.

TABLE 1. COVARIANCE FILTER PROPERTIES FOR VARIOUS WEIGHTS (N)

σ_d = 100 Hz and PRF = 5000 Hz						
	Improvement I(σ) (dB)			Frequency Response H(f) (dB)		
N	I(50)	I(100)	I(150)	HDB(500)	HDB(1500)	HDB(2500)
3	51	39	33	-16	1	4
4	66	60	48	-27	-5	5
5	84	78	61	-37	-2	6
6	91	85	67	-42	-3	6
7	75	69	53	-29	0	5
8	82	77	60	-35	1	5
9	99	96	73	-48	1	-2

values for $\{h_i\}$ and two frequency responses in decibels, one extending from DC to 5 σ_d, the second to PRF/2.

c. <u>Conventional Filters</u>

Although it is impossible to realize an ideal HPF, there are many reasonable approximations which emphasize such features as maximally flat passband (Butterworth), linear phase response (Bessel), or maximum stopband attenuation with specified passband ripple (Chebyshev). These models can be realized using infinite (IIR) and/or finite (FIR) impulse response designs. The IIR filters typically require recursive structures [5,6] and are not the subject of this report; rather, the design procedure to be considered in Section 3 utilizes an FIR structure with a Chebyshev error approximation to the H(f) of an ideal HPF. A procedure is described for minimizing the lower edge of the passband (PASSF) while maintaining a design specification in terms of passband ripple (RPB) and stopband attenuation (ASB) for a specified stopband (STOPF). The number of filter weights (NFILT) is upper bounded by the minimum allowable integration gain. The FIR filter, sometimes called a transversal filter, uses a nonrecursive structure and provides a frequency characteristic which typically is designed with linear phase and an amplitude response which is symmetric about PRF/2. As such, it is identical in structure with the covariance filter, but utilizes its weights to achieve a specified balance between ASB and RPB.

3. Chebyshev Design Procedure

The MTI design technique recommended for removal of clutter is based on the Chebyshev approximation to an ideal HPF. The algorithm, proposed is a streamlined version* of the optimum FIR linear phase digital filter program designed by McClellan, et al. [7]. It employs the Remez exchange algorithm to design a filter with minimum error between the actual and desired frequency response. The HPF design algorithm is coded as program MTI and is described in Appendix C.

a. Design Parameters

The MTI program requires values for the following parameters:

1) NFILT - Number of filter weights (taps or multipliers)
2) PASSF - Lower edge of the passband (Hz).
3) STOPF - Upper edge of the stopband (Hz).
4) WEIGHT - Ratio of passband error (δ) to stopband error (Δ).
5) RATIO - Ratio of Δ to the error at STOPF.
6) PRF - Pulse repetition frequency (Hz).
7) NEG - Symmetry parameter; (=0) if NFILT odd, (=1) if NFILT even.
8) LGRID - Controls density of grid points.

Some of these parameters are dependent upon the designer, while others are dependent upon design problem data, e.g., the clutter bandwidth. The relationship of the first six parameters to filter amplitude response is illustrated in Figure 4. Each parameter will be addressed separately in the following paragraphs.

The number of filter weights (NFILT ≤ 150) is selected by the designer. Typically, more weights yield better approximations to an ideal HPF, but the resultant design is more expensive and requires the radar to transmit at least NFILT pulses per beam dwell. System design considerations usually place a lower limit on IG; however, it follows from Equation (1) that increasing the number of weights has an adverse effect on IG. Consequently, an upper bound is established on NFILT and a trade-off between improved frequency response and IG must be considered.

---

*The original program contained options to design multiband filters, Hilbert transformers, and differentiators.

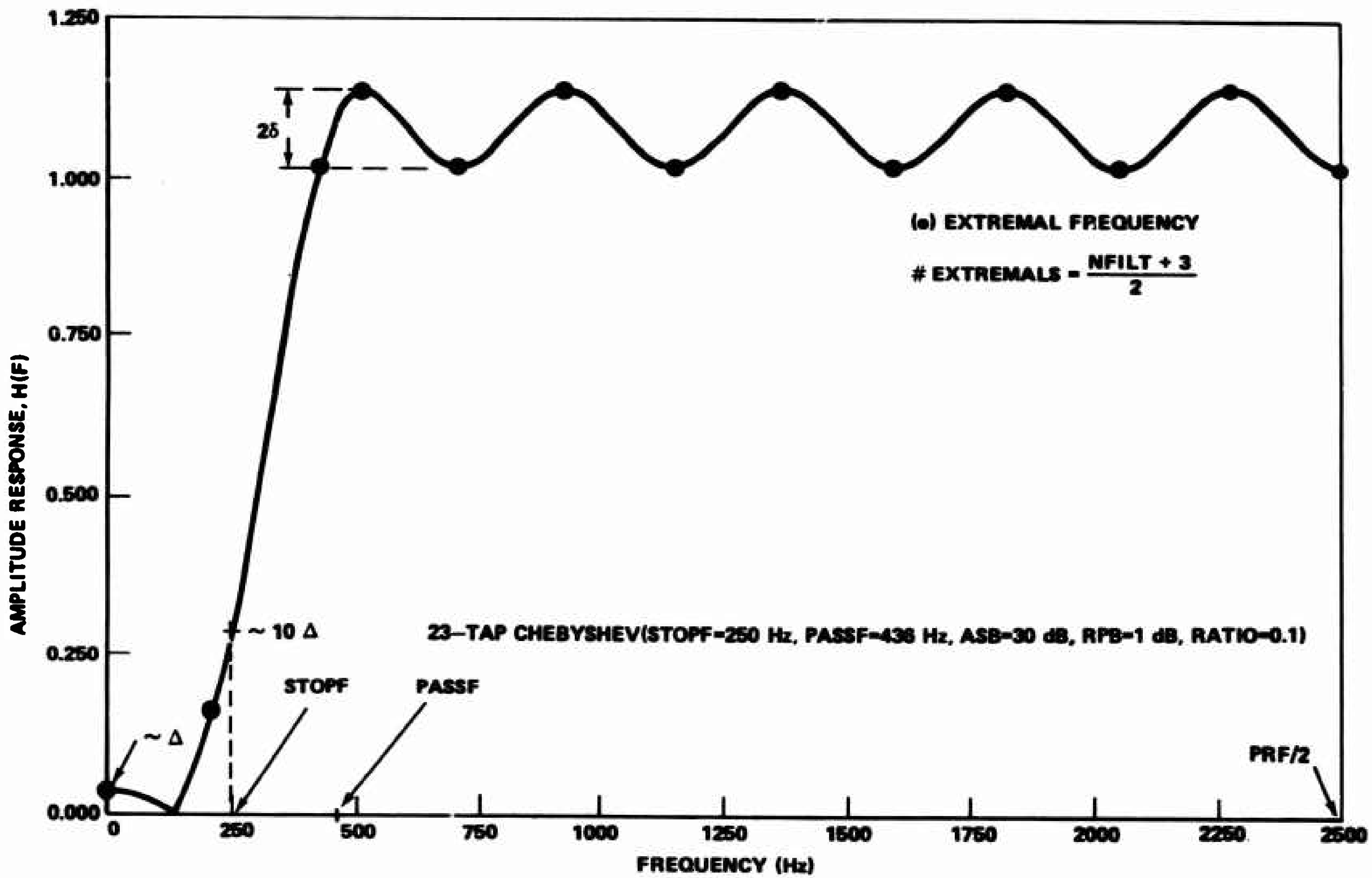

Figure 4. Effect of MTI design parameters on amplitude response.

The frequency response is improved by making the passband lower edge (PASSF) as small as possible to broaden the range over which doppler frequencies can be detected. For a given value of NFILT, the corresponding minimum value of PASSF consistent with the other design parameters can be obtained from a search algorithm MTIDSN which is described in Appendix D. It is assumed hereafter that PASSF is to be optimized for a given set of filter specifications.*

The stopband cutoff (STOPF) is typically determined by knowledge of the clutter spectrum. If the clutter is truly band limited, then STOPF equals this bandwidth. However, if the spectrum is exponentially decaying, then some approximate value is chosen; e.g., if the clutter spectrum is Gaussian, then STOPF = 2 to 4σ, where σ is the presumed clutter standard deviation. The value chosen for STOPF is dependent upon the desired I. The value of PASSF is relatively insensitive to small changes in the value of STOPF.

The ratio of the passband/stopband error (WEIGHT) is determined by first selecting the allowable RPB,

$$\text{RPB} = 20 \log\left[\frac{1+\delta}{1-\delta}\right] \text{ dB} \quad , \qquad (3)$$

where δ is the maximum passband error, and the ASB,

$$\text{ASB} = -20 \log\left[\frac{\Delta}{1+\delta}\right] \text{ dB} \qquad (4)$$

where Δ is the nominal stopband error,** and WEIGHT = δ/Δ. The value of WEIGHT is generated by program MTIDSN which includes RPB and ASB as two of its input parameters. The value of RPB is made as large as possible since larger RPB implies smaller transition bandwidth for a given NFILT. However, RPB must be consistent with the need to detect a doppler signal at frequency f_d anywhere in the passband, i.e., the frequency response in decibels, HDB(f_d) must exceed the minimum allowable value HPB_m which approximates -RPB/2. The value of ASB is specified in accordance with the desired I and type of stopband response. The value of PASSF is relatively insensitive to the value of ASB, but is quite dependent upon the value of RPB.

---

*If system considerations do not limit NFILT, then it is possible to estimate the value of NFILT required to achieve a specified PASSF using program ESTTAP which is described in Appendix E.

**The extreme deviations, $1 \pm \delta$ and $\Delta/W(f)$, occur at frequencies designated as extremal frequencies in Figure 4. The number of such extremals is (NFILT + 3)/2.

The uniformity of stopband response is controlled by the parameter RATIO which is the ratio W(STOPF)/W(DC), where W(f) is a linear function of frequency in the stopband. W(f) provides a means for placing relative emphasis on the attenuation in the stopband. Maximum emphasis occurs at DC, with uniform emphasis throughout if RATIO = 1, or triangular emphasis if RATIO < 1. The latter scaling can be useful when the clutter power density is known to be exponentially decaying, e.g., Gaussian. The effect of RPB, ASB, and RATIO = 0.1 on the frequency response of a 15-tap Chebyshev filter is shown in Figure 5. The other parameters shown will be described in Section 3.c.

The remaining three parameters PRF, NEG, and LGRID can be selected without much consideration on the part of the filter designer. The PRF is typically given to the designer and is based on considerations other than MTI signal processor design. For the MTI filter design, the symmetry parameter NEG = 0 if NFILT is odd and NEG = 1 if NFILT is even. In general, there are four possible combinations of NFILT and NEG which control the filter frequency response, i.e.,

$$H_i(f) = Q_i(f) \sum h_i(k) \cos(2\pi kf/T) \qquad i=1,2,3,4 \quad . \tag{5}$$

If NFILT is odd and NEG = 0, (i = 1), then $Q_1(f) = 1$, whereas if NFILT is even and NEG = 1 (i = 2), then $Q_2(f) = \sin(\pi fT)$. Either of these frequency shapes is acceptable for approximating a HPF; however, the other two combinations yield $Q_3(f) = \cos(\pi fT)$ and $Q_4(f) = \sin(2\pi fT)$ both of which place undesired mulls (blinds) at half the PRF (1/2T) which is supposed to be in the middle of the passband. The frequency response at DC, H(DC) is zero for even values of NFILT and equals $\sum h_i(k)$ for odd values. Finally, the grid density parameter (LGRID) must satisfy the inequality

$$(PRF/2)/STOPF < LGRID \times NFILT/2 < 1200 \quad , \tag{6}$$

where the minimum is determined by the desire to have at least one grid point in the stopband and the maximum is the dimension currently allowed various arrays in the program. Typically, LGRID is kept in the range 15 to 50 and does not produce any significant change in the resulting filter weights.*

*One exception is the case of a small, even value for NFILT and RATIO < 1.

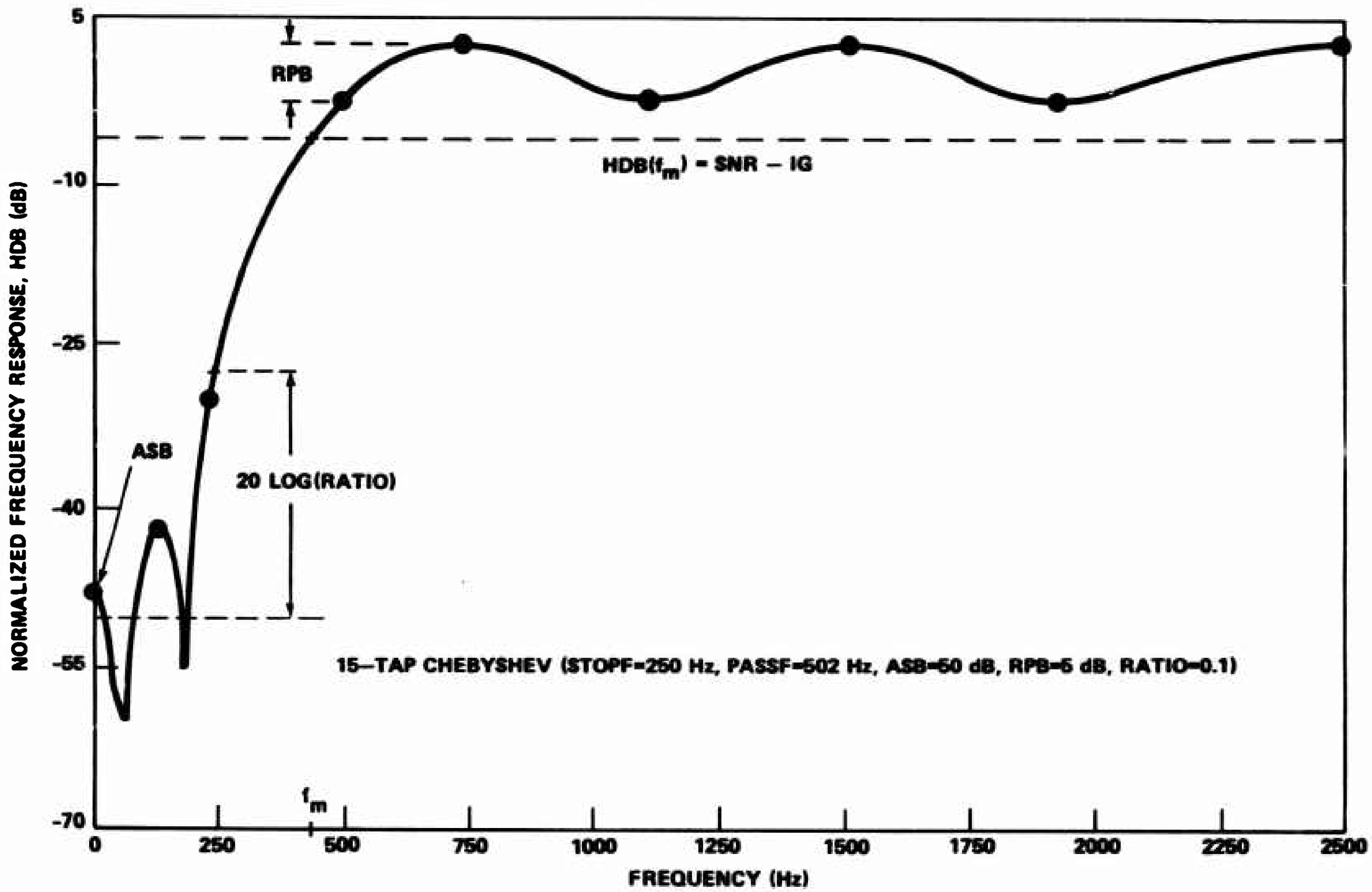

Figure 5. Effect of MTI design parameters ASB, RPB, and ratio on frequency response.

Specific guidelines for selecting the parameter values are given in the last two sections for narrowband and wideband clutter, respectively. Each section includes a detailed design example based on parameters associated with experimental array radar (EAR). Once the values of PASSF and WEIGHT are determined from program MTIDSN, the filter weights are obtained from program MTI. Using these weights, both frequency response and sensitivity studies are made. Minor adjustments are made to the design parameters and the procedure is reiterated as needed to obtain the final design.

b. EAR System Constraints

The EAR system currently has a PRF = 5 kHz and transmits 50 pulses per beam dwell, of which 48 are available for MTI signal processing. The system presently uses a fixed-window TPC and an integrator which sums 16 TPC outputs (R = 16). The integrator output is used to make a decision regarding the presence or absence of a target. Using Equation (1), the IG for R = 16 is 12.0 dB; consequently, using a smaller number of residues with a larger fixed-window MTI filter creates a trade-off between loss in integration gain and improved frequency response. The best values of NFILT are those for which NFILT $\times$ R $\sim$ 48. Typical combinations are 6 $\times$ 8, 12 $\times$ 4, 16 $\times$ 3, and 24 $\times$ 2 for NFILT even and 5 $\times$ 9, 15 $\times$ 3, or 23 $\times$ 2 for odd values.

Based on the 48-pulse constraint, the IG as a function of NFILT is shown in Table 2 and is used to select the candidate values for NFILT. It is evident that the loss in IG is no more than 5 dB when changing from 4 to 10 weights. The table can be used to estimate the loss in IG when using larger values of NFILT by recognizing that halving the number of outputs results in an additional 3 dB loss, e.g., with NFILT increased from 8 to 16 (R = 3), then IG = 7.8 - 3.0 = 4.8 dB.

TABLE 2. IG FOR EAR SYSTEM

NFILT	R	IG (dB)
3	16	12.0
4	12	10.8
5	9	9.5
6	8	9.0
7	6	7.8
8	6	7.8
9	5	7.0
10	4	6.0

The incoming pulse doppler signal is sampled by a nine-bit analog-to-digital converter (including sign) at a 5 MHz rate. Consequently, if a DC clutter component is suppressed to the extent that it does not influence the MTI output; i.e., within the analog-to-digital quantum interval, it is sufficient to reduce it by a factor 2^{-9}, which is equivalent to 54 dB attenuation. Details regarding the available processor input signal-to-thermal noise ratio (ISN) and the required output signal-to-noise ratio (OSN) to achieve a desired P_d for a specified P_f are discussed in Appendix F. It is assumed that the specification on I is sufficient to eliminate any clutter component in the MTI output. These constraints will be utilized in the following examples.

c. Narrowband Clutter Design

The design of an MTI filter to remove narrowband clutter is described in this section. The procedure is summarized by the flowchart given in Figure 6. It will be assumed that the narrowband clutter consists of a strong DC component which must be removed, i.e., attenuated by 54 dB, and that the AC clutter spectrum is Gaussian with $\sigma < 10$ Hz and the improvement needed is $I = 50$ dB. It is further stipulated that all targets which are closer than 8 km must be detected with $P_d = 0.5$ for $P_f = 10^{-6}$, which is shown in Appendix F to require OSN = 13 dB. It follows from Equation (7-2) that ISN = 7 dB; consequently, the MTI processor must provide the additional 6 dB required at the integrator output. Since the minimum passband response is $\simeq$ -RPB/2, i.e., the doppler signal is sometimes attenuated, it follows that IG > 6 dB and less than 10 weights must be used in constructing the MTI. It is desirable to provide the maximum passband in terms of the minimum frequency (f_m) for which

$$IG + HDB(f_m) = SNR\ dB \quad , \tag{7}$$

where $HDB(f_m)$ is the filter frequency response in dB at $f = f_m$ and SNR is the increase in signal-to-noise ratio required at the integrator output to insure a minimum probability of detection for a specified false-alarm rate. The location of f_m is illustrated in Figure 5 for SNR = 0 dB. It is evident that RPB could be increased by several decibels without crossing this boundary. Both IG and HDB(f) depend on NFILT, and based on inspection of Table 2, it is determined that the most likely choices would be 5, 6, 8, or 9 weights. The value NFILT = 7 is rejected because of a poor trade between IG and HDB(f).

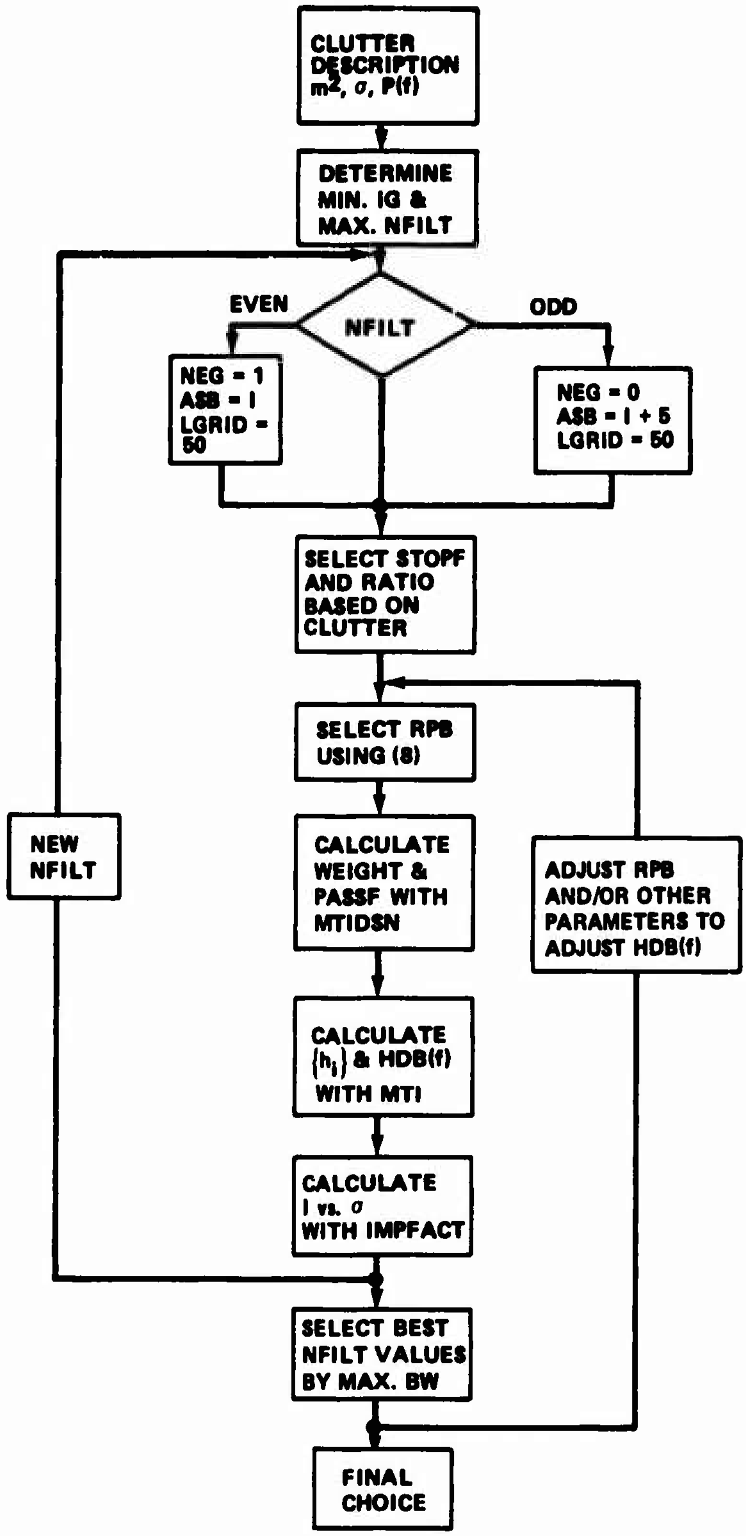

Figure 6. Design procedure for Chebyshev MTI filter.

The values of stopband attenuation (ASB) and bandwidth (STOPF) are selected to achieve the desired 50-dB improvement over the range $\sigma < 10$ Hz with the design value $\sigma_d = 5$ Hz. STOPF is set equal to $4\sigma_d = 20$ Hz initially, and ASB = I dB for NFILT an even value (6 or 8) and I + 5 dB for an odd value (5 or 9). A larger ASB for NFILT odd is required because the DC response HDB(DC) < -54 dB to satisfactorily eliminate the DC clutter component. However, such a constraint is not required for even values of NFILT since HDB(DC) = $-\infty$. Because of the narrow stopband, uniform weighting (RATIO = 1) is employed* and the grid density LGRID is set to 50.

The value of PASSF is relatively insensitive to changes in STOPF or ASB, both of which are important factors in determining I. The small variation of PASSF with STOPF is demonstrated in Table 3 for 9-tap and 10-tap Chebyshev filters with ASB = 50 dB, RPB = 4 dB, and uniform weighting. The variation of PASSF with ASB is shown in Figure 7. For very small or very large values of ASB, it is apparent that the 10-tap filter is superior; however, for most practical ground clutter designs requiring between 40-dB and 60-dB improvement, the 9-tap design is superior. The rapid decrease in PASSF for small values of ASB with the 10-tap design is explained by the null at DC which is sufficient to provide the necessary attenuation. However, the reason for the rapid rise in PASSF with ASB > 6^ dB for the 9-tap, but not the 10-tap, design is not obvious. Nevertheless, for MTI filters designed for ground clutter ($I \simeq 50$ dB), experience indicates that the odd values of NFILT provide a lower PASSF value.

TABLE 3. EFFECT OF STOPF ON PASSF FOR NARROWBAND-CLUTTER MTI (ASB = 50 dB, RPB = 4 dB, RATIO = 1)

STOPF (Hz)	PASSF (N = 9)	PASSF (N = 10)
5	426.4	507.1
10	425.4	544.0
15	424.4	554.3
20	426.5	559.3
25	425.5	561.6
30	424.5	564.0
35	426.5	566.3
40	470.4	568.7

*Triangular weighting (RATIO = 0.1) produced no change in weights for odd values of NFILT. Although it did produce a significant improvement in bandwidth for small, even values of NFILT, the resulting bandwidth was roughly comparable to that achieved with corresponding odd values.

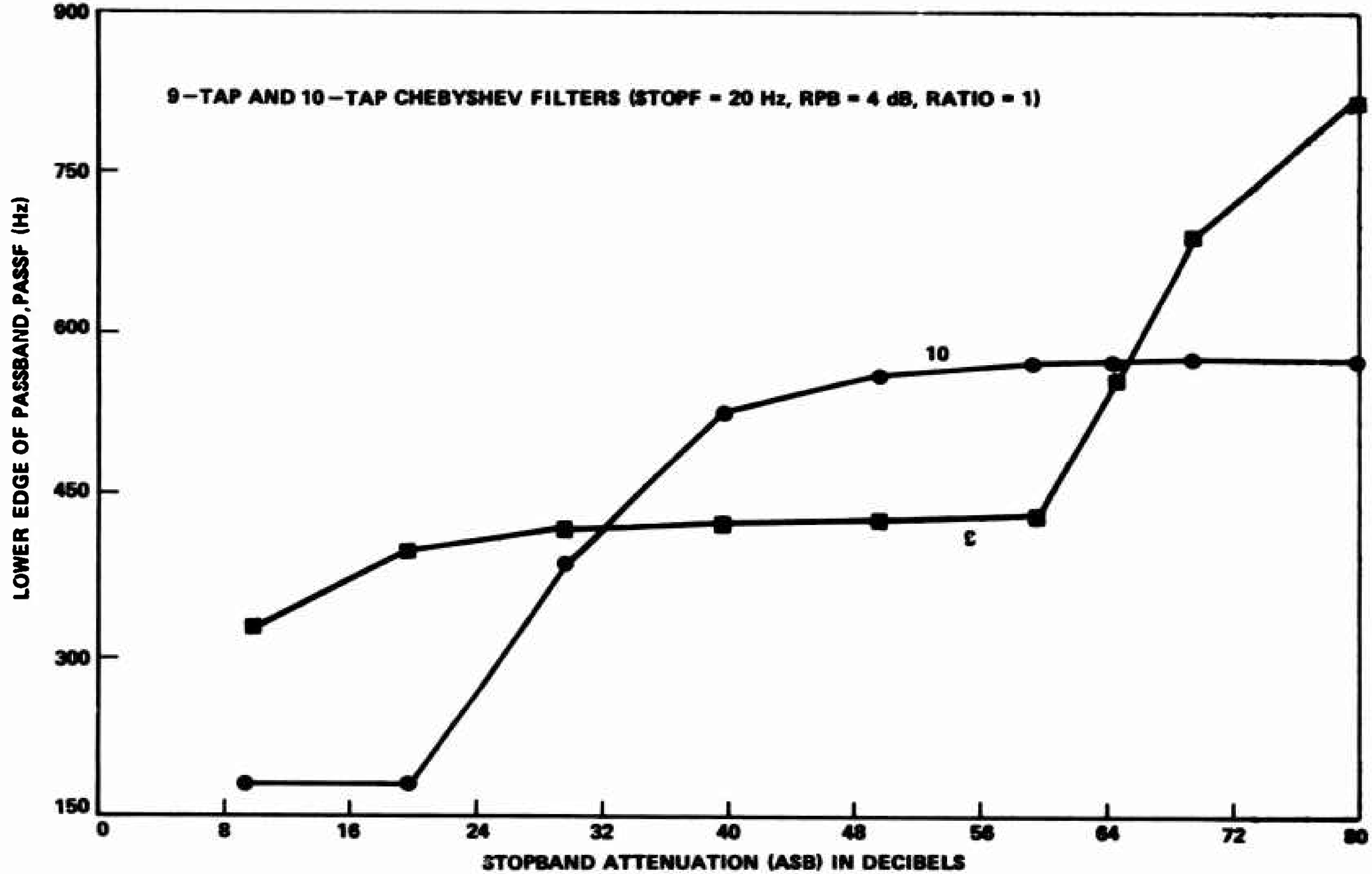

Figure 7. Effect of stopband attenuation (ASB) on PASSF.

RPB is set to the largest value consistent with Equation (7). Due to the difference in IG, RPB is respecified for each value of NFILT, with the initial estimate for RPB given by

$$RPB \simeq 2[IG - SNR] \quad . \tag{8}$$

In the derivation of Equation (8), it was assumed that $HDB(f_m) = -RPB/2$; however, because of the normalization applied to the filter weights, RPB is only approximately symmetric and is biased positive for most values. The values of $HDB(f_m)$ consistent with Equation (7) and values of RPB which satisfy Equation (8) for various NFILT are shown in Table 4.

TABLE 4. RPB CONSISTENT WITH SNR = 6 dB (IG, RPB, AND HDB ARE MEASURED IN dB; f_m AND BW IN Hz)

NFILT	IG	RPB	$HDB(f_m)$	f_m	BW
5	9.5	7.0	-3.5	631	3738
6	9.0	6.0	-3.0	806	3388
8	7.8	3.6	-1.8	685	3630
9	7.0	2.0	-1.0	517	3966

Once the values of NFILT and the other filter parameters have been selected, corresponding values of PASSF and WEIGHT can be obtained from program MTIDSN, after which the actual weights $\{h_i\}$ are obtained from program MTI. These weights plus the design parameters are then supplied to program IMPFACT which plots the frequency response shown in Figure 8 and sensitivity study shown in Figure 9. It is apparent that the nine-tap design has a superior frequency response and more than sufficient I; however, it is necessary to consider the effect of lower integration gain on bandwidth before reaching a final conclusion. Using Equation (7), it follows that the bandwidth over which the probability of detection is satisfied is given by

$$BW = PRF - 2f_m \quad . \tag{9}$$

The value of f_m for each NFILT is obtained from the normalized frequency responses shown in Figure 8. The resulting bandwidths for these designs are also given in Table 4. It is evident that the nine-tap solution provides an additional 228-Hz bandwidth, or 4.5% of the total PRF, over the next leading contender. Although this appears to be a most satisfactory design, some further improvement is possible by altering ASB or

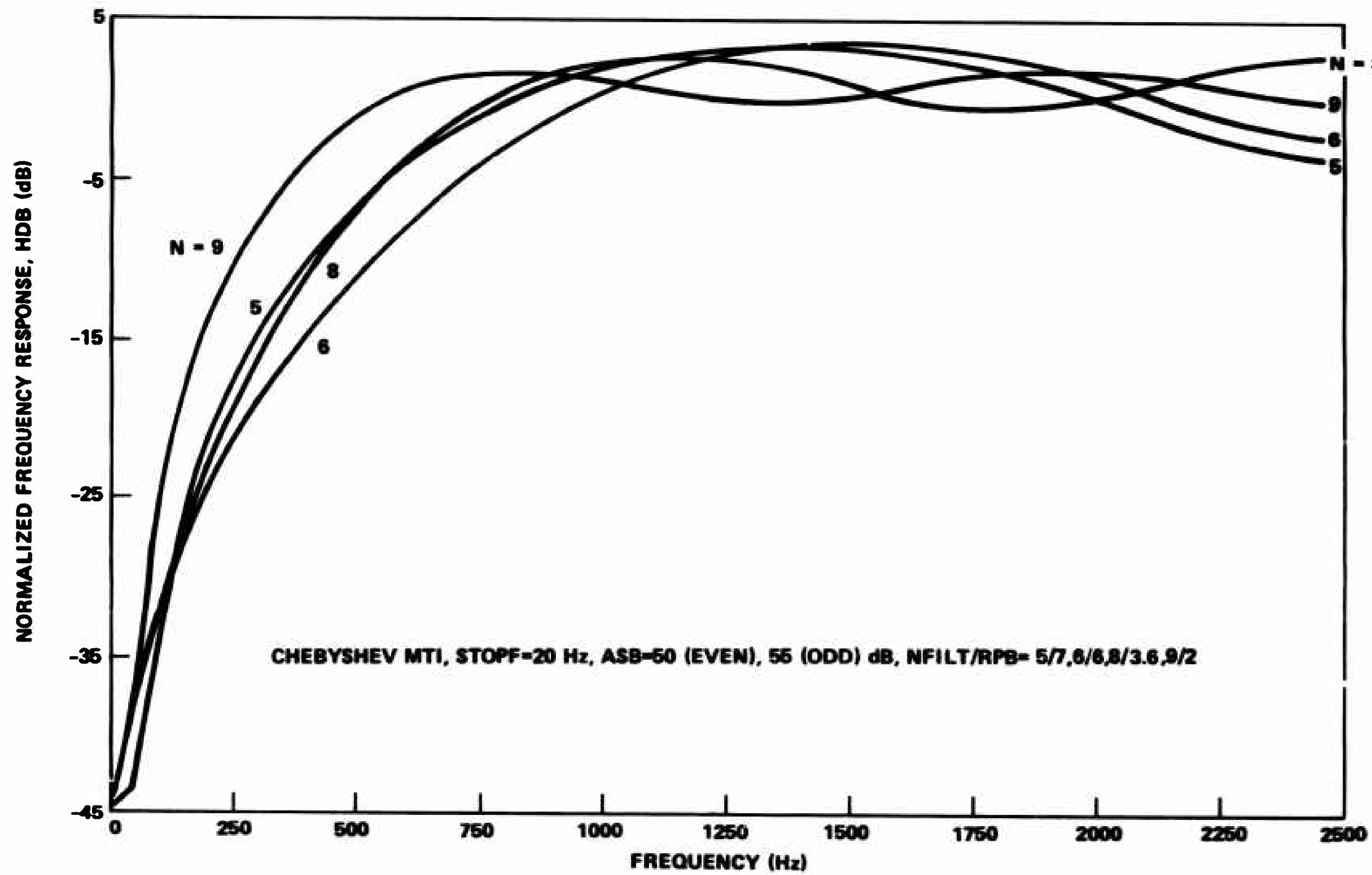

Figure 8. Frequency responses for narrowband-clutter filters.

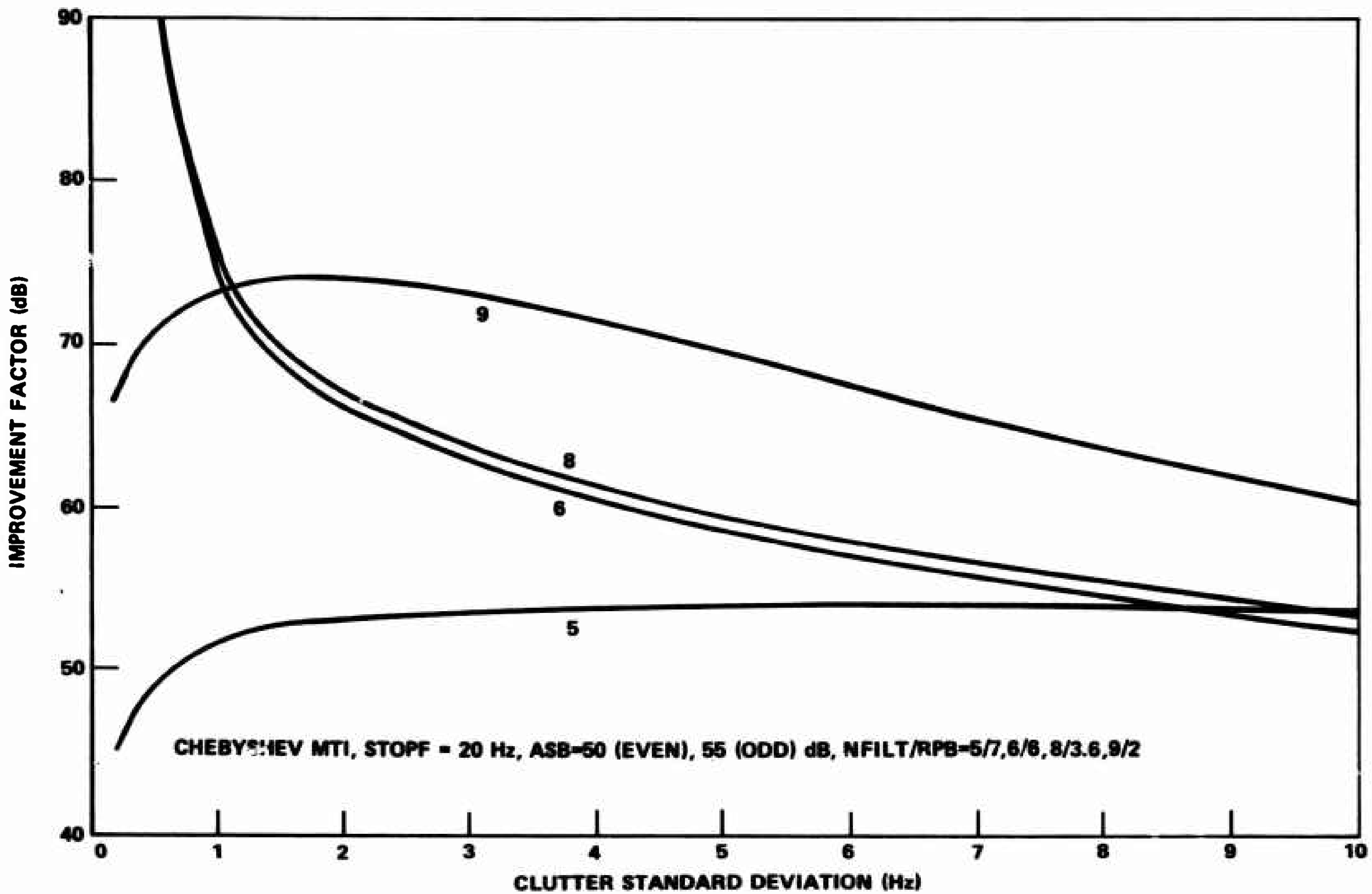

Figure 9. Sensitivity studies for narrowband-clutter designs.

RPB. Attempts to decrease ASB to 50 and 45 dB for the nine-tap filter reduced I without improving H(f). However, increasing RPB from 2 to 3 dB reduced f_m from 517 Hz to 480 Hz, which increased the usable BW an additional 1.5% without lowering I below 50 dB. The minimum response in the passband (HPB_m) was -1.0 dB.

In summary, it would appear that significant improvement in usable bandwidth can be obtained by replacing the TPC or covariance (COV) design with an NFILT-tap Chebyshev (CHEB) filter without reducing I below acceptable limits. Comparative passband and I data for various designs are given in Table 5. It is evident that even the five-tap CHEB design represents an effective tradeoff between I and BW, gaining 9.6% additional bandwidth over the COV design while retaining 54 dB of I.

TABLE 5. SUMMARY OF NARROWBAND-CLUTTER MTI DESIGNS (SNR = 6 dB)

Design	NFILT	IG	HDB(f_m)	BW	I(σ = 5)
TPC	3	12.0	-6.0	3132	92
COV	4	10.8	-4.8	3380	87
	5	9.5	-3.5	3412	85
CHEB	5	9.5	-3.5	3738	54
	6	9.0	-3.0	3388	58
	8	7.8	-1.8	3630	59
	9	7.0	-1.0	4040	69

If the input signal is adequate to provide the desired P_d and a sufficient number of pulses are transmitted, then it appears that the MTI filter should be designed with large values of RPB and NFILT. This follows from Figures 10 and 11 which clearly demonstrate the monotonic decrease in PASSF with increases in either of these parameters. Furthermore, an odd number of taps provides more bandwidth than a comparable even number, although the distinction disappears for NFILT > 20.

d. <u>Wideband Clutter Design</u>

The design procedure for wideband clutter is described in Figure 6 and is similar to that for narrowband clutter. The primary differences are the use of triangular emphasis in the stopband (RATIO = 0.1) and the corresponding selection of ASB = I + 10 dB to account for the decreased attenuation near STOPF. It is assumed that the wideband clutter is Gaussian (σ = 100 Hz) without a steady-state component and the clutter power is considerably less than the ground clutter example. Consequently, I = 20 dB is considered adequate and STOPF is set equal to 2.5σ versus 4σ for the narrowband case because

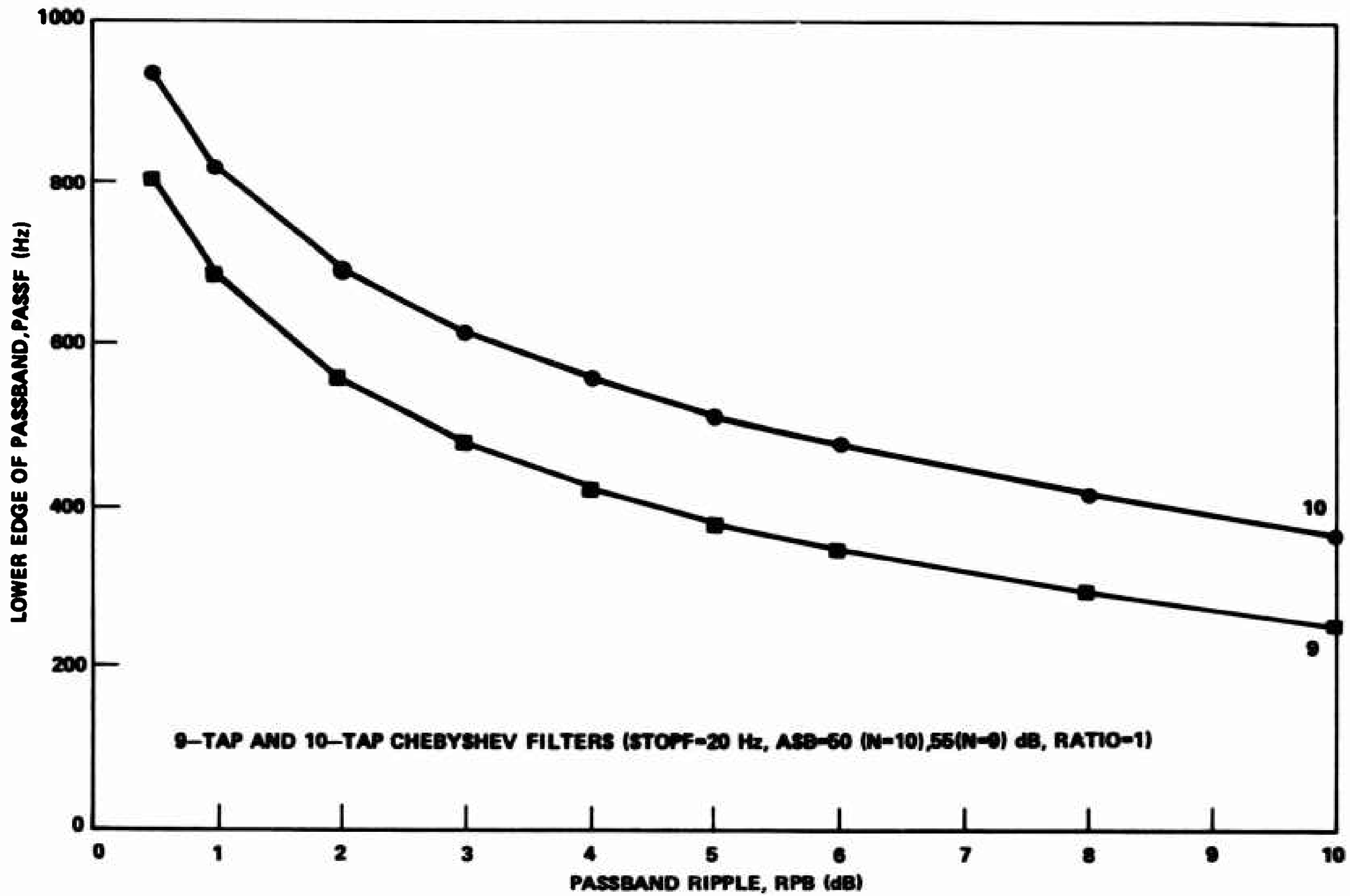

Figure 10. Effect of varying RPB on PASSF.

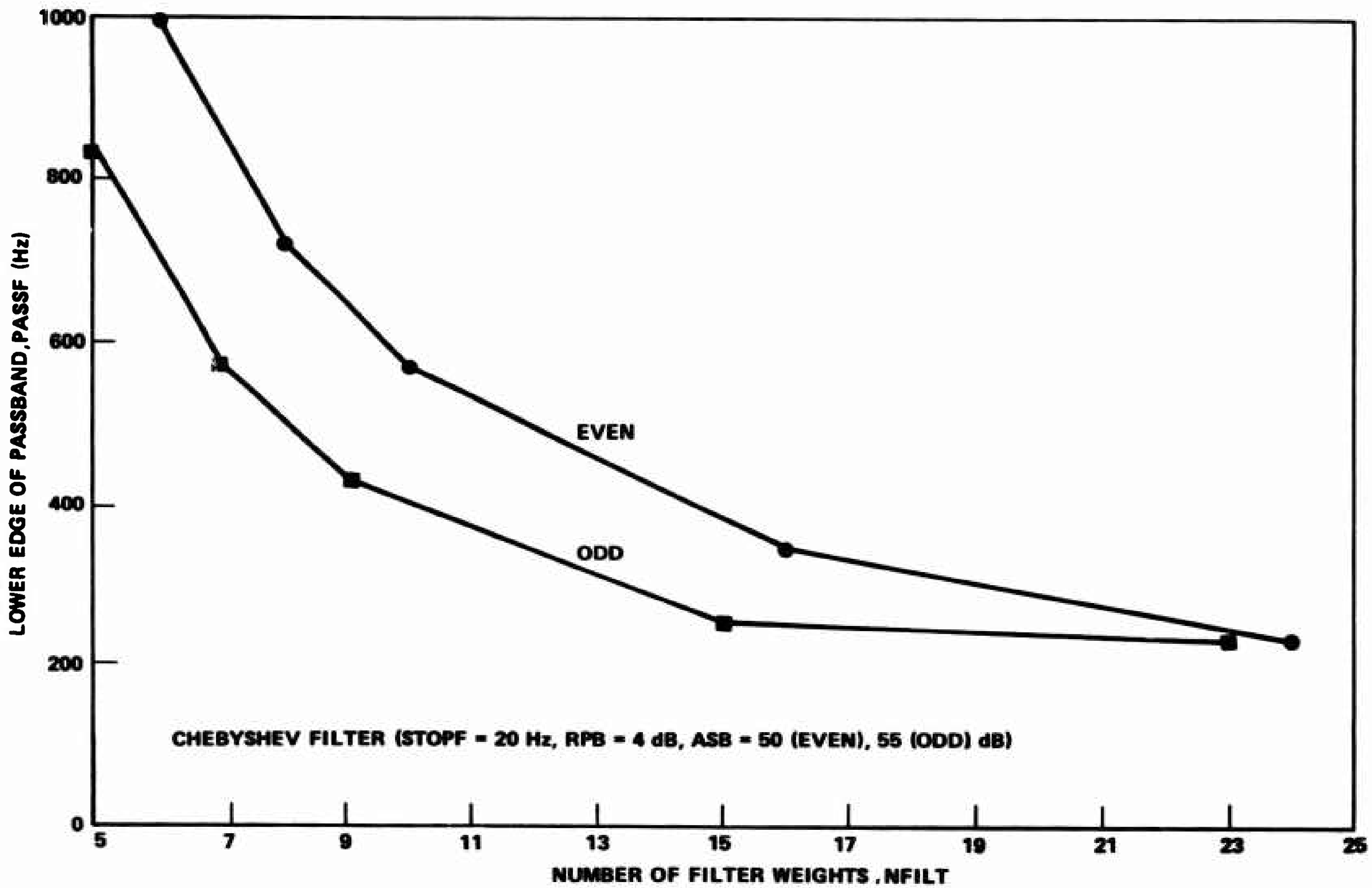

Figure 11. Effect of even versus odd number of weights on PASSF.

of the significant reduction in required I. It is further assumed that the maximum range of interest is 5 km (ISN = 15 dB) and that a target must be detected on the first scan with a probability $P_d = 0.8$, given a false-alarm probability of 10^{-5}. Consequently, the OSN = 18 dB and the signal processor must provide a minimum of 3 dB gain.

The I specification (I = 20 dB) is met by letting ASB = 30 dB, STOPF = 250 Hz, and RATIO = 0.1. The SNR = 3 dB requirement is satisfied by letting the initial RPB estimate be determined from

$$\mathrm{RPB} = 2[\mathrm{IG} - 3] \ \mathrm{dB} \quad . \tag{10}$$

It follows from an extension of Table 2 that NFILT < 25 for IG > 3 dB. The effect of using a triangular emphasis, rather than uniform, in the stopband is demonstrated in Table 6 for several values of NFILT with RPB selected to satisfy Equation (10). To make a comparison, the value of ASB is lowered from 30 dB for RATIO = 0.1 to 25 dB for uniform emphasis. In every instance the passband is larger, i.e., f_m is lower, for triangular emphasis rather than uniform. It is also apparent that HDB(f) is biased such that the passband ripple is not symmetric with respect to 0 dB. For the lower values of NFILT with large RPB estimates, the minimum decibel response in the passband (HPB_m) is less than $\mathrm{HDB}(f_m)$, which means that RPB must be decreased. Conversely, for larger values of NFILT, the value of HPB_m is more than $\mathrm{HDB}(f_m)$; in fact, it is even greater than 0 dB for the 23-tap and 24-tap designs. Actually, if this were not the case, it would have been impossible to design a 23-tap or 24-tap filter since it follows from Equation (10) that the initial estimate of RPB should be 0 dB, rather than the 1 dB used for these cases, a value which cannot be used in the Chebyshev algorithm. The reason that the bias is more pronounced for wideband clutter filters than it was for the narrowband design can be explained in terms of the noise power gain normalization of the coefficients. Since the objective is to provide 0 dB gain to the broadband white noise, it follows that the passband noise must be amplified since the fraction of the noise in the stopband (2 STOPF/PRF) is attenuated by ASB decibels.

The second estimate for RPB is determined by doubling the difference HPB_m-$\mathrm{HDB}(f_m)$ and adding this factor to the original RPB estimate. For example, with NFILT = 8, the difference is -1.2 dB, which when doubled and added to the original estimate (9.6 dB) yields 7.2 dB. In all of the remaining cases, except nine taps, the new value of RPB is larger than the original which implies better passbands than the first design. In addition, designs for NFILT = 11 and 12 are included since they represent additional IG versus NFILT tradeoffs, i.e., four outputs or IG = 6 dB. An initial estimate of RPB = 6 dB was selected

TABLE 6. INITIAL DESIGNS FOR WIDEBAND CLUTTER EXAMPLE (RATIO = 0.1, ASB = 30 FOR T, RATIO = 1.0, AND ASB = 25 FOR U)

NFILT	IG	RPB	HDB(f_m)	Triangular (T)		Uniform (U)	
				HPB_m	f_m	HPB_m	f_m
8	7.8	9.6	-4.8	-6.0	465	-5.9	532
9	7.0	8.0	-4.0	-4.8	387	-4.5	538
15	4.8	3.6	-1.8	-1.4	432	-1.3	491
16	4.8	3.6	-1.8	-1.5	365	-1.3	458
23	3.0	1.0	0.0	0.1	419	0.2	459
24	3.0	1.0	0.0	0.1	429	0.3	488

for these designs using Equation (10). The new design results are shown in Table 7. It is evident that the largest useable bandwidth is associated with NFILT = 16, and that 9 and 12 weights are almost as good, yielding a 1.5% decrease in useable bandwidth. It is also evident from inspection of the HPB_m columns in Tables 6 and 7 that the adjustment of RPB tended to overcompensate for the original error between HPB_m and HDB(f_m). However, it follows from inspection of the f_m columns in the two tables that the changes in f_m were less than 25 Hz, i.e., 1% change in the useable bandwidth.

TABLE 7. SECOND-ITERATION DESIGNS FOR WIDEBAND CLUTTER EXAMPLE (RATIO = 0.1, ASB = 30 dB, AND STOPF = 250 Hz)

NFILT	IG	RPB	HDB(f_m)	f_m	HPB_m
8	7.8	7.2	-4.8	483	-3.9
9	7.0	6.4	-4.0	396	-3.5
11	6.0	6.0	-3.0	420	-3.1
12	6.0	6.0	-3.0	397	-3.1
15	4.8	4.4	-1.8	420	-2.0
16	4.8	4.2	-1.8	360	-1.9
23	3.0	1.2	0.0	422	0.0
24	3.0	1.2	0.0	422	0.0

A third adjustment in RPB is made to the leading candidates (NFILT = 9, 12, and 16) and the resulting design results are shown in Table 8. Frequency responses for these designs are shown in Figure 12 and the corresponding sensitivity studies (I versus σ) in Figure 13. Although the value of I is adequate for σ < 100 Hz, it follows from inspection of Figure 13 that the value of σ could not increase more than 10 Hz before I < 20 dB. This example illustrates that the tradeoff between IG and NFILT is not obvious and that a thorough study of potential candidates is needed. Moreover, unlike the narrowband design example, odd values of NFILT are not necessarily superior to corresponding even values, e.g., 15 versus 16 taps, and triangular emphasis can be effectively employed to reduce PASSF.

TABLE 8. FINAL DESIGN RESULTS FOR WIDEBAND CLUTTER EXAMPLE

NFILT	IG	RPB	$HDB(f_m)$	f_m	HPB_m
9	7.0	7.0	-4.0	394	-4.0
12	6.0	5.8	-3.0	396	-3.0
16	4.8	4.0	-1.8	364	-1.8

4. Conclusions

It has been demonstrated that conventional MTI design procedures concentrate on removing clutter, but fail to maximize the passband throughout which doppler signals can be detected with an acceptable probability. The effective manner in which the Chebyshev design accomplishes the desired trade-off between reduced I and increased passband throughout which the MTI gain requirement is met has been illustrated for a variety of examples including both narrowband and wideband clutter. Although the designs were oriented toward the EAR system, the procedure is quite general and can be applied to any system which utilizes a fixed-window MTI; however, a similar design procedure could be employed for systems which use moving-window MTI filters. The effect of pulse-to-pulse or block PRF stagger has not been studied. The recent work of Ewell [8] in this area of constraining improvement while maintaining a flat passband response for both staggered and uniform PRF appears to serve the same end; however, the means are quite different. Unfortunately, the available information [8] is inadequate to make any meaningful comparisons between the two procedures. Further coordination could prove to be useful, particularly if pulse staggering is contemplated.

Results would tend to indicate that enough weights should be employed to achieve a reasonably flat passband without seriously degrading the integration gain. Furthermore, the passband ripple

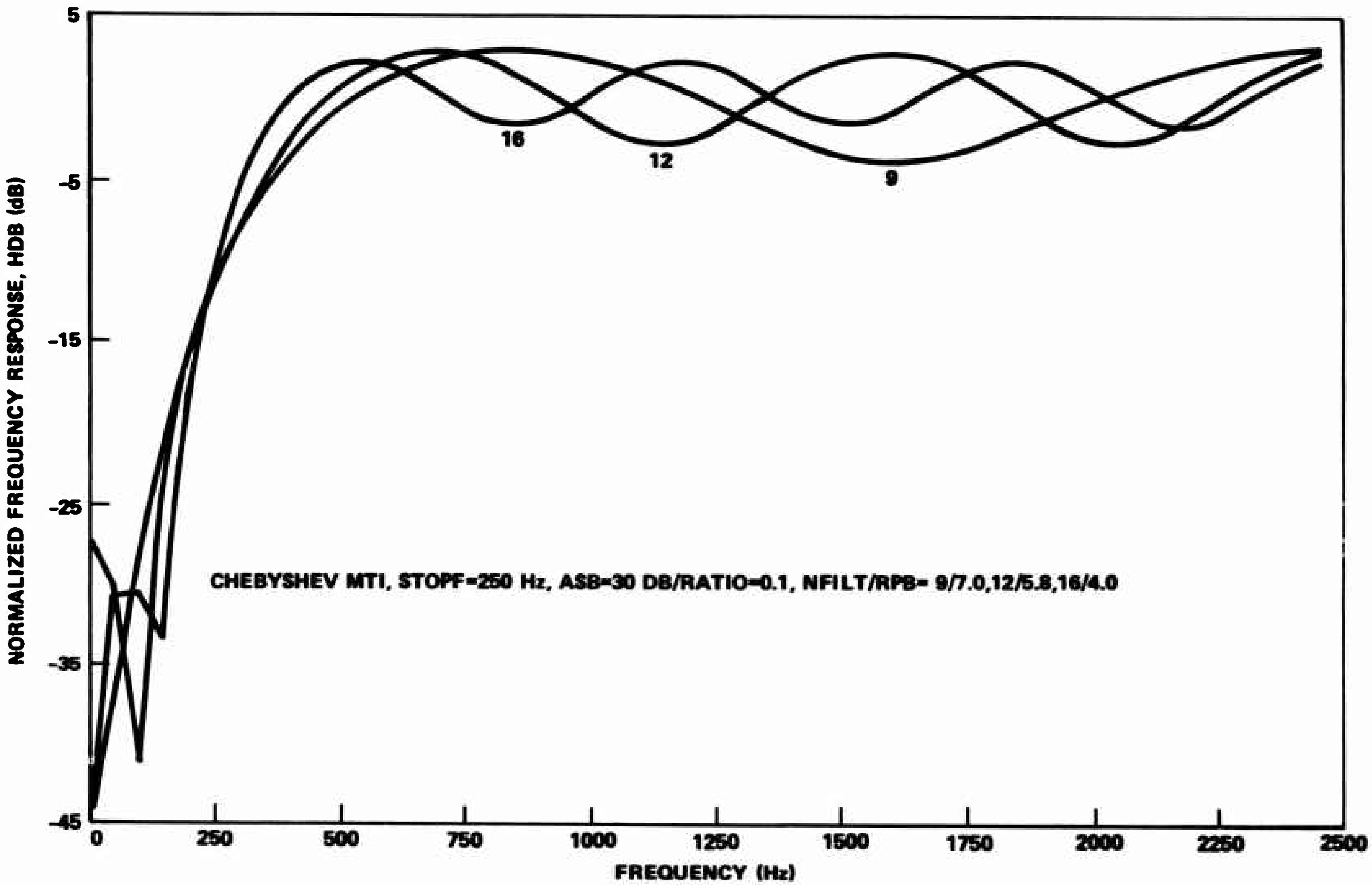

Figure 12. Frequency responses for wideband-clutter (σ = 100 Hz) filters.

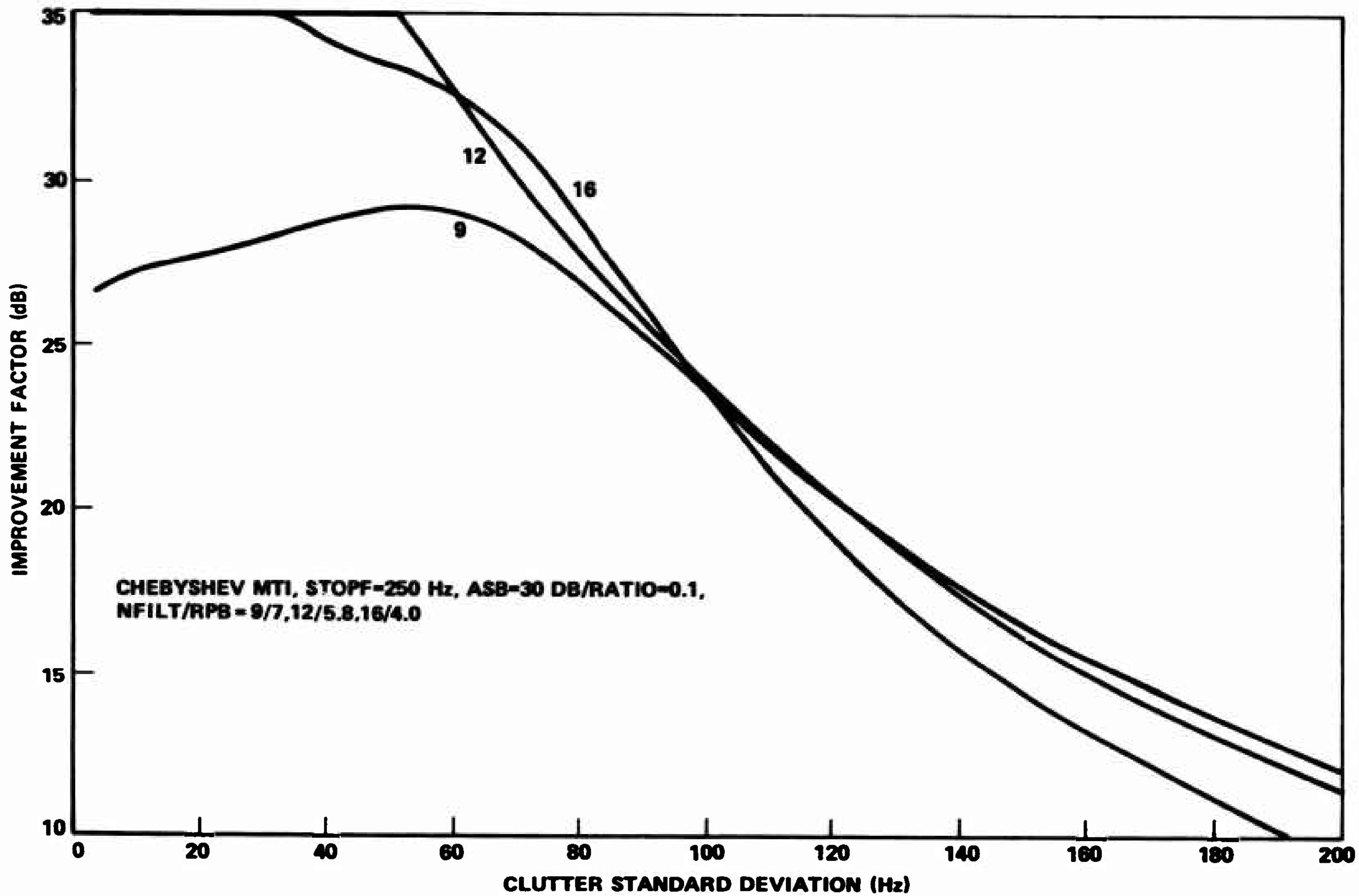

Figure 13. Sensitivity studies for wideband-clutter designs.

should be made as large as possible consistent with MTI gain requirements. The clutter attenuation is controlled by selection of stopband attenuation and cutoff frequency, factors which can be varied considerably without significantly affecting the passband. Uniform emphasis is used for ground clutter designs and results indicate that the best designs are achieved with an odd number of weights; however, very narrowband clutter ($\sigma < 1$ Hz) would be most effectively handled with an even number of weights due to the null in the frequency response at DC. Triangular emphasis should be employed in broadband clutter designs, because this further improves the passband. Both even and odd values of NFILT must be examined for wideband clutter designs and several design iterations are required due to the passband gain associated with filter-weight normalization affecting the performance specifications.

Appendix A.

IMPFACT – PROGRAM TO COMPUTE MTI IMPROVEMENT FACTOR

This program estimates the MTI filter I by computing the average signal gain (S) and clutter gain (C). The clutter gain is defined as the ratio of output clutter power to input clutter power,

$$C = 2\int_0^{f_t} P(f)\,|H(f)|^2\,df \quad , \tag{A-1}$$

where f_t = PRF/2, H(f) is the frequency response of the MTI filter and P(f) is the normalized clutter power-density spectrum, which is assumed to be either Gaussian or uniform. In either case, the standard deviation (σ_d) is supplied by the user, and the upper limit is replaced by $f_t = 4\sigma_d$ for Gaussian or $\sqrt{3}\,\sigma_d$ for uniform clutter. The numerical approximation to Equation (A-1) for Gaussian clutter becomes

$$C = \frac{2}{\sqrt{2\pi}\sigma_d} \sum_{j=1}^{U} |H(f_j)|^2 \exp[-(f_j/\sigma_d)^2/2]\,\Delta f \quad , \tag{A-2}$$

where $f_1 = 0$, $\Delta f = f_{j+1} - f_j$, and $U = 4\sigma_d/\Delta f$. The average signal gain (S) is defined as the ratio of output to input signal power

$$S = \frac{2}{P_i}\int_{f_p}^{f_t} K\,|H(f)|^2\,df \quad , \tag{A-3}$$

where the input power (P_i) is assumed to be uniformly distributed over the passband, i.e., $P_i = K\,[f_t - f_p] = K\,[PRF/2 - 3\sigma_d]$. Consequently, S can be approximated by

$$S = \frac{1}{N}\sum_{j=P}^{T} |H(f_j)|^2 \quad , \tag{A-4}$$

where $T = f_t/\Delta f$, $P = f_p/\Delta f$, and $N = T - P + 1$. The frequency spacing (Δf) is typically selected to yield between 500 and 2000 samples equally distributed between DC and PRF/2. I is defined as

$$I = 10\log(S/C)\ \text{dB} \quad . \tag{A-5}$$

The integral approximations are admittedly simplistic; however, the error is typically less than one dB from theoretical values. Moreover, the program was designed to compare several filter designs and the error can be presumed to affect each in the same manner. A sensitivity study is performed for up to 50 values of σ spaced in increments of the initial value (CLUINC).

The listing for program IMPFACT is given in Table A-1. The listing begins with explanations of the data card parameters required. A typical run time is 2 seconds for a 10-tap filter and 50 σ-values. The program utilizes subroutines for a line-printer plot (GRAPH1) or X-Y plotter (PLT) to present the output curves for the frequency response and sensitivity study. These routines are not presented with the listing for IMPFACT as they are rather lengthy and are replaceable by similar routines typically available for a particular computer system.

TABLE A-1. PROGRAM IMPFACT LISTING

```
      PROGRAM IMPFACT   (INPUT,OUTPUT,TAPE5,TAPE6,TAPE1)
      DIMENSION Z(49),FREQ(2001),AMPSQ(2001),CLUVAR(50),FACI(50),LC(8),
     1         LX(4),LY(3),LA(2),LB(4), LZ(8) ,H(2001),HS(50,5),F(50)
C*********************************************************************
C** PROGRAM TO COMPUTE THE IMPROVEMENT FACTOR (FACI) IN DB BY FIRST   **
C** CALCULATING THE SIGNAL GAIN (SIGGAIN) AND CLUTTER GAIN (CLUGAIN). **
C** THE CLUTTER GAIN IS COMPUTED FOR SIGMA = CLUINC TO FSIGMA         **
C** USER INSERTS CARD  - NRUNS =  - WHICH INDICATES THE NUMBER OF RUNS**
C** DATA SUPPLIED BY 3 CARDS PRIOR TO FILTER DATA FOR EACH OF NRUNS   **
C**    FIRST CARD IS LABEL (LC) FOR THE FREQUENCY RESPONSE.           **
C**    SECOND CARD IS A LABEL (LZ) FOR THE SENSITIVITY PLOT.          **
C**    THIRD CARD CONTAINS VARIOUS CONTROL PARAMETERS - -             **
C**    1. NFILT - NUMBER OF CURVES / PLOT (.LE.5).                    **
C**    2. NP    - THE NUMBER OF SIGMA AND FREQUENCY PTS. (.LE.50).    **
C**    3. IPLOT - (=0) X-Y PLOT / TAB.DATA, (=1) GRAPH1, (=2) NO PLOT. **
C**    4. AMIN  - MINIMUM IMPROVEMENT FACTOR (DB) FOR X-Y PLOT.       **
C**    5. AMAX  - MAXIMUM IMPROVEMENT FACTOR (DB) FOR X-Y PLOT.       **
C**    6. FRINC - THE SPACING BETWEEN FREQUENCY SAMPLES.  PRESENTLY   **
C**               LIMITED TO 2001 SAMPLES BETWEEN DC AND PRF/2        **
C**    7. SIGMA - THE ASSUMED SIGMA (HZ) FOR WHICH FILTER DESIGNED    **
C**    8. CLUINC- THE CLUTTER STD. DEV. INCREMENT AND INITIAL VALUE.  **
C**    9. FSIGMA- THE LARGEST CLUTTER STD. DEV. (HZ).                 **
C**   10. ICPD  - CLUTTER POWER DENSITY - GAUSSIAN(=0), UNIFORM(=1)   **
C** USER DATA SUPPLIED BY 2 CARDS FOR EACH FILTER INCLUDED IN A RUN - **
C**    1. PARAMETERS USED TO DESIGN FILTER + NOISE POWER GAIN (PGN)   **
C**            (SEE FORMAT 30 FOR DESCRIPTION OF 8 PARAMETERS)        **
C**    2. THE SET OF FILTER COEFFICIENTS (Z) (SEE FORMAT 40)          **
C*********************************************************************
      DATA  PI2/6.283185308/,SQTPI/2.506628275/
C  ABSCISSA AND ORDINATE LABELS FOR FREQUENCY AND SENSITIVITY PLOTS
      DATA LA/10H FREQUENCY,10H (HZ) *   /
      DATA LB/10HNORMALIZED,10H FREQUENCY,10H RESPONSE,,10H HDB (DB)*/
      DATA LX/10H CLUTTER S,10HTANDARD DE,10HVIATION (H,10HZ) *      /
```

TABLE A-1. (Continued)

```
      DATA LY/10H IMPROVEME,10HNT FACTOR ,10H(DB) *
      READ(5,10) NRUNS
   10 FORMAT(I2)
      DO 200 IRUN=1,NRUNS
      READ(5,15) LC,LZ
   15 FORMAT(8A10)
      READ(5,25) NFILT,NP,IPLOT,AMIN,AMAX,FRINC,SIGMA,CLUINC,FSIGMA,ICPD
   25 FORMAT(3I3,6F5.1,I2)
C  MOVE DO-LOOP AFTER FORMAT 20 IF IPLOT = 2 AND NRUNS .GE. 2
      DO 90 J=1,NFILT
      WRITE(6,20) LC
   20 FORMAT(1H1, 20X, 8A10 // * NTAPS  FSTOP  FPASS  WEIGHT  RATIO  PRF
     1  SYM  PGN    Z(1)      Z(2)      Z(3)      Z(4)      Z(5)      Z(6)
     1  Z(7)      Z(8)      Z(9)   * / )
      READ(5,30) N, NEG, FSTOP, FPASS, PRF, RATIO, WT, PGN
   30 FORMAT(I3,I2,5X,F5.2,F10.5,F5.2,F5.2,10X,2F5.3)
      READ(5,40) (Z(I),I=1,N)
   40 FORMAT(17X,7F9.5 / (10F8.5))
      WRITE(6,50) N, FSTOP,FPASS,WT,RATIO,PRF,NEG,PGN,(Z(I),I=1,N)
   50 FORMAT(I4,2X,5F7.1,I3,F6.3,9F9.5 /(50X,9F9.5 /))
C  CALCULATE SUMMATION FOR A GIVEN FREQUENCY
C
      JPASS = 0
      IFINAL = 1 + IFIX(PRF /(2.0 * FRINC) )
      DO 2 K = 1, IFINAL
      XREALS = 0.0
      XIMAGS = 0.0
      AA = 0.0
      DO 1 I = 1, N
      XREAL = Z(I) * COS(PI2 * AA * FREQ(K) / PRF)
      XIMAG= -Z(I) * SIN(PI2 * AA * FREQ(K) / PRF)
      XREALS = XREALS + XREAL
      XIMAGS = XIMAGS + XIMAG
    1 AA = AA + 1.0
      AMPSQ(K) = XREALS**2 + XIMAGS**2
```

TABLE A-1. (Continued)

```
      IF(AMPSQ(K).LE.1.0E-10) AMPSQ(K) = 1.0E-10
      H(K)    = 10.0*ALOG10(AMPSQ(K)) - PGN
      IF(H(K)   .GE.0.0) JPASS = JPASS + 1
    2 FREQ(K+1) = FREQ(K) + FRINC
C COMPUTE DOPPLER GAIN FROM 3*SIGMA TO PRF/2
      GAIN = 0.0
      IPASS = IFIX(3.* SIGMA / FRINC) + 1
      DO 4 K = IPASS,IFINAL
    4 GAIN = GAIN + AMPSQ(K)
      AVGGAIN = GAIN / FLOAT( IFINAL - IPASS + 1)
      SIGAIN =  10.0*ALOG10(AVGGAIN) - PGN
      DO 55 L = 1,NP
      K = (L-1)*(IFINAL-1)/NP + 1
      F(L)=FREQ(K)
   55 HS(L,J) = H(K)
C  SENSITIVITY STUDY - VARY SIGMA FROM CLUINC TO FSIGMA IN STEPS = CLUINC
      IF(IPLOT.EQ.1) GO TO 5
      WRITE(6,60) SIGMA,JPASS,IFINAL
   60 FORMAT(1HO, *  SENSITIVITY STUDY FOR SIGMA = *,F6.2, * HZ. ALL GAI
     1NS NORMALIZED WRT NOISE GAIN. FRACTION.GE.0 DB= *,I4,*/*,I4,//
     2 *  SIGMA(HZ)   IMPROVEMENT(DB)   CLUTTER GAIN(DB)  SIGNAL GAIN(DB
     3)     FREQ(HZ)      H(F)-SQD.(DB) * / )
    5 DO 9 I=1,NP
      CLUVAR(I) = CLUINC * FLOAT(I)
      CLUTTER - 0.0
      IF(ICPD .EQ. 1) GO TO 6
C  COMPUTE CLUTTER GAIN FROM DC TO 4 * SIGMA IF GAUSSIAN SPECTRUM
      DENEXP = 2.0 * CLUVAR(I)**2
      ISTOP =  IFIX(4.0*CLUVAR(I)/FRINC) + 1
      DO 3 K = 1,ISTOP
    3 CLUTTER = CLUTTER + AMPSQ(K) * EXP(-FREQ(K)**2/DENEXP)
      CLUTPOW = 2.0 * CLUTTER * FRINC /( SQTPI * CLUVAR(I) )
      GO TO 8
```

TABLE A-1. (Concluded)

```
C  COMPUTE CLUTTER GAIN FROM DC TO SQRT(3) * SIGMA IF UNIFORM SPECTRUM
 6   ISTOP = 1 + IFIX(1.732050808 * CLUVAR(I) / FRINC)
     DO 7 K = 1,ISTOP
 7   CLUTTER = CLUTTER + AMPSQ(K)
     CLUTPOW = CLUTTER / FLOAT(ISTOP)
 8   IF(CLUTPOW.LE.1.0E-10) CLUTPOW = 1.0E-10
     CLUGAIN = 10.0*ALOG10(CLUTPOW) - PGN
     FACI(I) = SIGAIN  - CLUGAIN
     IF(IPLOT.EQ.1) GO TO 9
     WRITE(6,70) CLUVAR(I),FACI(I),CLUGAIN,SIGAIN,F(I),HS(I,J)
70   FORMAT(3X,F5.1,3(10X,F8.3),8X,F6.1,8X,F8.3)
 9   CONTINUE
     IF(IPLOT.EQ.0) GO TO 85
     IF(IPLOT.EQ.2) GO TO 90
     WRITE(6,80) SIGAIN, JPASS,IFINAL
80   FORMAT(1H0,* X = CLUTTER SIGMA (HZ) VS. Y = IMPROVEMENT FACTOR (DB
    1) WITH DOPPLER GAIN = *,F6.3,* DB AND SIGNAL GAIN FRACTION =*,I4,
    2  */*,I4 )
     CALL GRAPH1(CLUVAR,FACI,NP,1)
     GO TO 90
85   MP = - NP
     IF(J.EQ.1) MP = NP
     CALL PLT(CLUVAR,FACI,MP, 0 ,LX,LY,LZ,0.,FSIGMA,AMIN,AMAX)
90   CONTINUE
     IF(IPLOT.NE.0) GO TO 200
     HMIN = -45.
     HMAX = 5.
     FUP = 2500.
95   DO 100 M = 1,NFILT
     MP = - NP
     IF(M.EQ.1) MP = NP
100  CALL PLT(F,HS(1,M),MP,0,LA,LB,LC,0.,FUP,HMIN,HMAX)
200  CONTINUE
     CALL RSTR(2)
     STOP
     END
```

Appendix B.

OPTMTI – MTI FILTER DESIGN PROGRAM USING COVARIANCE METHOD

This program utilizes an optimization algorithm suggested by Capon [4] for the design of an MTI filter. Computer listings for the main program and subroutines OPTWT, JACXM, and AMPSQDB are found in Tables B-1 through B-4, respectively. The program is designed on the premise that the clutter power-density spectrum is zero-mean Gaussian with the standard deviation (SIGMA) supplied by the user, who also supplies the parameters PRF and NTAPS. Consequently, the autocorrelation and covariance function are also Gaussian. The format and restriction on these parameters are listed in the comment cards found at the beginning of the program listing. The design permits the inclusion of pulse-to-pulse stagger in the specifications, in which case the PRF is replaced by the desired blind speed in hertz (BSHZ). The second data card includes the stagger ratios $\{R(i)\}$ required, which are all equal to 1.0 for an unstaggered design. Details of pulse-to-pulse stagger are found in the literature, e.g. Skolnik [9]. The main program handles the input-output processing including calls to the aforementioned subroutines and GRAPH1. Subroutine OPTWT computes the desired covariance matrix which is passed to JACXM where the associated eigenvalues and eigenvectors are computed. The desired set of weights $\{D\}$ are obtained in OPTWT by selecting the eigenvector associated with the minimum eigenvalue. Subroutines JACXM and OPTWT were supplied by Raytheon with a modified version of the main program. Subroutine AMPSQDB generates the frequency response in dB (HDB) for a transversal FIR digital filter with a set of specified weights $\{Z\}$. Designs can be computed for NTAPS ≤ 15; however, there is no assurance that I increases monotonically with NTAPS for SIGMA/PRF < 0.03. Degenerate solutions are obtained for NTAPS > 6 with SIGMA/PRF < 0.002. Execution time for a given design is typically less than 1 second.

TABLE B-1. PROGRAM OPTMTI LISTING

```
      PROGRAM OPTMTI(INPUT,TAPE5,OUTPUT,TAPE6)
      DIMENSION FREQ(500),HDB(500),DELAY(15),TAU(15),D(15),R(14),T(15)
C***********************************************************************
C** PROGRAM TO DESIGN OPTIMUM MTI TRANSVERSAL FILTERS WHEN THE CLUTTER**
C** IS GAUSSIAN WITH STD. DEVIATION = SIGMA (HZ). THE DESIGN IS BASED **
C** ON THE COVARIANCE CONCEPT PROPOSED BY J. CAPON IN IEEE TRANS. ON   **
C** INFORMATION THEORY, APRIL 1964, PP. 152-159. PROGRAM IS A MODIFIED**
C** VERSION OF THE RAYTHEON PROGRAM OPTMTI WHICH COMPUTES STAGGERED    **
C** TIME DELAYS T(I) FROM USER DATA FOR UNSTAGGERED PRF (HZ) AND STAG-**
C** GER RATIOS R(I)                                                   **
C** INPUT CARDS. (PRECEDE DATA CARDS BY NDATA (I2) = NUMBER OF RUNS.  **
C** 1. THREE PARAMETERS                                               **
C**       A. NTAPS - NUMBER OF FILTER COEFFICIENTS (I2), MAX. OF 15.  **
C**       B. PRF   -  PULSE REP. FREQ. (HZ) OF UNSTAGGERED SYSTEM     **
C**       C. SIGMA - GAUSSIAN CLUTTER STD. DEVIATION IN HERTZ (F5.2)  **
C** 2. STAGGER RATIOS R(I),I=1 TO 14 (F6.2)                           **
C***********************************************************************
      PI = 3.14159265
      READ(5,5) NDATA
    5 FORMAT(I2)
      DO 90 IR=1,NDATA
      READ(5,6) NTAPS, PRF, SIGMA
    6 FORMAT(I2,F8.2,F5.2)
      COFEXP = 1.41421356 * PI * SIGMA
      NN= NTAPS-1
      READ (5,10)  (R(I),I=1,NN)
   10 FORMAT (14F6.2)
      AVGR = 0.
      DO 11 I = 1,NN
   11 AVGR = AVGR + R(I)
      AVGR = AVGR / FLOAT(NN)
      BSHZ = PRF * AVGR
      T(1)=0.00
      DO 15 I=2,NTAPS
```

TABLE B-1. (Continued)

```
   15 T(I)= R(I-1) / BSNZ
      CALL OPTWT(T,SIGMA,D,NTAPS)
      WRITE (6,20)   (R(I),I=1,NN)
   20 FORMAT(1H0,18X,* MTI FILTER DESIGNED BY COVARIANCE TECHNIQUE * //
     * * STAGGER RATIOS  *,15F5.0 /)
      WRITE (6,25)   (T(I),I=1,NTAPS)
   25 FORMAT (1H0,* TIME DELAYS*,7E15.7,/,(13X,7E15.7/))
      WRITE (6,30) SIGMA
   30 FORMAT(1H0,27HOPTIMUM WEIGHTS FOR SIGMA =,F7.2,* HZ*/)
      PGN= 0.00
      DO 35 I=1,NTAPS
      WRITE (6, 40) I, D(I)
   40 FORMAT(10X,4HWGT(,I2,3H) =,F10.5)
   35 PGN= PGN + D(I)**2
      DBN= 10.00 * ALOG10(PGN)
C     REMOVED FPEQUENCY OFFSET (FREQ) LOOP 12. OMEGA = 0.
C     REMOVED SIGMA (GIG) LOOP 13.
C     REMOVED FREQUENCY (FRAG) LOOP 17. WO = 0.
      CLU=0.00
      JMAX= NTAPS
      IMAX= NN
      TAUSUM= 0.00
      DO 45 I=1,JMAX
      TAU(I)= T(I+1)
      TAUSUM= TAUSUM + TAU(I)
   45 DELAY(I)= TAUSUM
      DO 50 IC=1,IMAX
      IA=IC+1
      DO 50 JC=IA,JMAX
      TOP=0.00
```

TABLE B-1. (Concluded)

```
      JA=JC-1
      DO 55 KC=IC,JA
   55 TOP=TOP+R(KC) / BSHZ
   50 CLU = CLU + 2.* D(IC) * D(JC) * EXP(-(COFEXP*TOP)**2)
      CLU = CLU + PGN
      IF(CLU) 60,60,70
   60 WRITE (6,65)
   65 FORMAT(15X,15HIMPOSSIBLE CASE)
      GO TO 80
   70 CLUDB= 10.00 * ALOG10(CLU)
      GAI = DBN - CLUDB
      WRITE (6,75) DBN, CLUDB, GAI
   75 FORMAT (15H0 NOISE GAIN = ,F7.2,17H  CLUTTER GAIN = ,F7.2,5X,
     1 *  IMPROVEMENT = *,F7.2,* (ALL IN DB)* )
   80 CONTINUE
      EPLON = 1.E-10
      FFREQ = 5.* SIGMA
      NP = 51
      DO 85 J=1,2
      CALL AMPSQDB(D,NTAPS,NP,0.,FFREQ,PRF,EPLON,DBN,HDB,FREQ)
      WRITE(6,84)
   84 FORMAT(1H0,* NORMALIZED AMPLITUDE (DB) VS. FREQUENCY (HZ) *)
      CALL GRAPH1 (FREQ,HDB,NP,1)
      NP = 201
      EPLON = 1.E-5
   85 FFREQ = PRF / 2.
   90 CONTINUE
      STOP
      END
```

TABLE B-2. SUBROUTINE OPTWT LISTING

```
      SUBROUTINE OPTWT(S,SIGMA,D,N)
      DIMENSION S(15), A(15,15), B(15,15), COVMX(15,15), D(15)
      Q = 3.14159265
      CONS = 2.*Q**2
      CK = -CONS*SIGMA**2
      SI = 0.00
      DO 10 I=1,N
      SI = SI+S(I)
      SJ = 0.00
      DO 10 J=1,N
      SJ = SJ+S(J)
      COVMX(I,J) = EXP(CK*(SI-SJ)**2)
   10 A(I,J) = COVMX(I,J)
      WRITE (6,100)
  100 FORMAT (1H1,* COVARIANCE MATRIX *,//)
      DO 200 K=1,N
      WRITE (6,101)  (A(K,KJ),KJ=1,N)
  101 FORMAT(1H ,15F8.4)
  200 CONTINUE
      ACC=1.E-6
      IT=1000
      CALL JACXM(A,B,N,ACC,IT,1)
      WRITE (6,105)
  105 FORMAT (1H-,* EIGENVALUES *,//)
      DO 450 L=1,N
      WRITE (6,102)  (A(L,JL),JL=1,N)
  102 FORMAT(1H ,10E13.3)
  450 CONTINUE
      WRITE (6,110)
  110 FORMAT (1H-,* EIGENVECTORS SCALED*,//)
      DO 550 L=1,N
      WRITE (6,101)  (B(L,JL),JL=1,N)
```

TABLE B-2. (Concluded)

```
 550 CONTINUE
     AMIN=A(1,1)
     ICOL=1
     DO 1000 JJ=2,N
     IF(A(JJ,JJ).GE.AMIN) GO TO 1000
     AMIN=A(JJ,JJ)
     ICOL=JJ
1000 CONTINUE
     DO 1001 LL=1,N
     D(LL)=B(LL,ICOL)
1001 CONTINUE
     RETURN
     END
```

TABLE B-3. SUBROUTINE JACXM LISTING

```
      SUBROUTINE JACXM(A,E,N,ACC,IT,L)
      DIMENSION A(15,15),E(15,15)
C                    -----------------------------------------------
C                    INITIALIZATION - IND=1 IMPLIES A TRANSFORMATION
C                    IS INITIATED.  -    E(I,J) IS THE TRANSFORMATION
C                    MATRIX INITIALLY SET TO THE UNIT MATRIX
C                    -----------------------------------------------
      GO TO (1,2),L
    1 DO 6 I=1,N
      DO 5 J=1,N
    5 E(I,J)=0.00
    6 E(I,I)=1.00
C                    -----------------------------------------------
C                    COMPUTATION OF INITIAL NORM
C                    -----------------------------------------------
2     NI=0
      IND=0
      VF=ACC
      VI=0.00
      DO 10J=2,N
      K=J-1
      DO 10 M=1,K
   10 VI=VI+2.*A(M,J)*A(M,J)
      VI=SQRT(VI)
      IF(L.EQ.1) VF=VI*ACC/N
      IF(L.EQ.1) V=VI
   14 V=V/N
      IF(V-VF) 110,15,15
   15 DO 100 J=2,N
      K=J-1
      DO 90 M=1,K
      IF(ABS(A(M,J))-V) 90,20,20
```

TABLE B-3. (Continued)

```
C                   ------------------------------------------------
C                   INITIATE TRANSFORMATION
C                   ------------------------------------------------
   20 IND= 1
      NI= NI+1
C                   ------------------------------------------------
C                   COMPUTATION OF SINE AND COSINE
C                   ------------------------------------------------
      Y=-A(M,J)
      U=0.5*(A(M,M)-A(J,J))
      IF(A(M,M)-A(J,J)) 22,21,22
   21 U=ABS(U)
   22 W=Y/SQRT(Y*Y+U*U)
      IF(U.LT.0.00) W=-W
      S=W/SQRT(2.*(1.+SQRT(1.-W*W)))
      C=SQRT(1.-S*S)
C                   ------------------------------------------------
C                   TRANSFORMATION OF THE M-TH AND J-TH
C                   COLUMNS AND ROWS
C                   ------------------------------------------------
      DO 50 I=1,N
      IF(I.EQ.M.OR.I.EQ.J) GO TO 45
      A(I,M)=A(I,M)*C-A(I,J)*S
      A(I,J)=A(M,I)*S+A(I,J)*C
      A(M,I)=A(I,M)
      A(J,I)=A(I,J)
C                   ------------------------------------------------
C                   COMPUTATION OF THE EIGENVECTORS
C                   ------------------------------------------------
   45 SS=E(I,M)
      E(I,M)=E(I,M)*C - E(I,J)*S
   50 E(I,J)=SS*S+E(I,J)*C
      SAM=A(M,M)
      A(M,M)=A(M,M)*C*C+A(J,J)*S*S-2.*A(M,J)*S*C
```

TABLE B-3. (Concluded)

```
      A(J,J)=SAM*S*S+A(J,J)*C*C+2.*A(M,J)*S*C
      A(M,J)=0.00
      A(J,M)=A(M,J)
      IF(NI.EQ.IT) GO TO 109
   90 CONTINUE
  100 CONTINUE
      IF(IND.EQ.0) GO TO 14
      IND=0
      GO TO 15
  109 ACC=VF
      WRITE (6,1004)
 1004 FORMAT(//1X,16HMAX. IT IN JACMX//)
      STOP
  110 IT=NI
      ACC=VF
      RETURN
      END
```

TABLE B-4. SUBROUTINE AMPSQDB LISTING

```
      SUBROUTINE AMPSQDB(Z,N,NP,SFREQ,FFREQ,PRF,EPLON,PGN,HDB,FREQ)
      DIMENSION Z(N),HDB(NP),FREQ(NP)
C*****************************************************************************
C**   ROUTINE CALCULATES FREQUENCY RESPONSE FOR TRANSVERSAL FILTER WITH     **
C**   N TAPS,Z(N),AND RETURNS THE AMP.SQUARED IN DB,HDB(NP),LESS THE        **
C**   NOISE POWER GAIN,PGN,FOR EACH OF NP FREQUENCIES,FREQ(NP), BACK TO     **
C**   THE CALLING PROGRAM.                                                  **
C**   PARAMETER DEFINITIONS ARE        Z = COEFFICIENT VALUES               **
C**                                    N = NUMBER OF COEFFICIENTS           **
C**                                    NP = NUMBER OF FREQUENCY POINTS      **
C**                                    SFREQ = STARTING FREQUENCY           **
C**                                    FFREQ = FINAL FREQUENCY              **
C**                                    PRF = PULSE REPETITION FREQUENCY     **
C**                                    EPLON= LOWER BOUNDARY FOR LOG ARGUEMENTS*
C**                                    PGN = NOISE POWER GAIN (DB)          **
C*****************************************************************************
      PI2 = 6.283185308
      FRINC = (FFREQ-SFREQ)/FLOAT(NP-1)
      FREQ(1) = SFREQ
      IF(EPLON.LE.1.0E-10) EPLON=1.0E-10
      DO 1 K=1,NP
      XREALS = 0.
      XIMAGS = 0.
      AA = 0.
C** CALCULATE SUMMATION FOR A GIVEN FREQUENCY
      DO 2 I=1,N
      XREAL  = Z(I)*COS(PI2*AA*(FREQ(K)/PRF))
      XIMAG =-Z(I)*SIN(PI2*AA*(FREQ(K)/PRF))
      XREALS = XREALS + XREAL
      XIMAGS = XIMAGS + XIMAG
    2 AA = AA + 1.
      AMPSQ = XREALS**2 + XIMAGS**2
C** CALCULATE DB RESPONSE
      IF(AMPSQ .LE. EPLON) AMPSQ = EPLON
      HDB(K) = 10.* ALOG10(AMPSQ) - PGN
    1 FREQ(K+1) = FREQ(K) + FRINC
      RETURN
      END
```

Appendix C.

MTI – PROGRAM TO SELECT CHEBYSHEV FILTER WEIGHTS

Program MTI is a digital filter design program based on the Chebyshev error algorithm [7] which is specialized to approximate an HPF by minimizing the maximum error between the actual and desired frequency response. The Remez exchange method is used to achieve a minimum weighted Chebyshev error in the frequency response approximation. The user provides eight parameters for each design. Six of these parameters were previously used by program MTIDSN (described in Appendix D) and the remaining two, WEIGHT and PASSF, were the output from MTIDSN. The same data card is used for both programs with MTIDSN parameters ASB and RPB bypassed in the MTI format. Details on how to select values for the input parameters were given in Section 3.

Output data include two sets of weights, one which meets the original design specifications, and a second which has been normalized to have zero decibel noise power gain. A normalized frequency response is provided between SFREQ and FFREQ using NP equally spaced values. A plot of the stopband emphasis W(f), which is either triangular or uniform, can be obtained by replacing statement 185 with CALL GRAPH1 (GRID, WT, IGRID, 1). A listing of program MTI is contained in Table C-1. Subroutines used by MTI include AMPSQDB, GRAPH1, and REMEZ. A listing for AMPSQDB is given in Table B-2 and GRAPH1 is a standard X-Y line printer routine. Subroutine REMEZ is detailed in Reference 7, with the only changes being the removal of double precision variables and minor changes in the dimensions of subscripted variables to conform with those in program MTI.

TABLE C-1. PROGRAM MTI LISTING

```
      PROGRAM MTI(INPUT,TAPE5,OUTPUT,TAPE6)
      COMMON PI2,AD,DEV,X,Y,GRID,DES,WT,ALPHA,IEXT,NFCNS,NGRID
      DIMENSION IEXT(79),AD(79),ALPHA(79),X(79),Y(79),H(79),EXTF(79)
     *,EDGE(20),FREQ(500),HDB(500),Z(150),DFS(1200),GRID(1200),WT(1200)
C***********************************************************************
C**   THIS PROGRAM IS USED TO DESIGN LINEAR PHASE FIR DIGITAL FILTERS **
C**   FOR USE IN MTI RADARS AND IS A MODIFIED VERSION OF THE MCCLELLAN**
C**   PROGRAM WHICH EAS PUBLISHED IN THE DECEMBER, 1973 9SSUE OF THE  **
C**   IEEE TRANSACTIONS ON AUDIO AND ELECTROACOUSTICS.                **
C**   MODIFICATIONS INCLUDE:                                          **
C**     1. LIMITED TO HIGHPASS FILTER DESIGN  (NTAPS.LE.150)          **
C**     2. INCLUDES SUBROUTINE AMPSQDB TO PLOT FREQUENCY RESPONSE     **
C**     3. NORMALIZES COEFFICIENTS W.R.T. TO SUM H(I)**2              **
C**     4. NORMALIZES FREQUENCY RESPONSE BY SUBTRACT NOISE GAIN (PGN) **
C**     5. FILTERS CAN BE DESIGNED WITH NEGATIVE SYMMETRY (N EVEN)    **
C**   (PRECEDE FIRST CARD BY NDATA = (I2), THE NUMBER OF RUNS.)       **
C**   I N P U T   D A T A   C O N S I S T S   O F   O N E   C A R D : **
C**     1. NFILT - NUMBER OF FILTER WEIGHT (.LE. 150)                 **
C**     2.  NEG  - COEFFICIENT SYMMETRY, POSITIVE(=0) OR NEGATIVE(=1) **
C**     3. LGRID - GRID DENSITY -- LGRID * (NFILT + 1)/ 2 .LE. 1200   **
C**                IF LGRID = 0, PROGRAM DEFAULTS TO MAXIMUM VALUE    **
C**     4. STOPF - UPPER EDGE OF STOPBAND (HZ)                        **
C**     5. PASSF - LOWER EDGE OF PASSBAND (HZ)                        **
C**     6.  PRF  - PULSE REPETITION FREQUENCY (HZ)                    **
C**     7. RATIO - RATIO OF WT(STOPF) / WT(DC)   (.LE. 1)             **
C**     8. WEIGHT- RATIO OF PASSBAND / STOPBAND RIPPLE                **
C***********************************************************************
      DATA PI/3.14159265359/,EDGE(1)/0./,EDGE(4)/0.5/
      PI2 = 6.28318530718
C     P R O G R A M   I N P U T   S E C T I O N .
      READ (5,60) NDATA
   60 FORMAT (I2)
      DO 500 IREAD=1,NDATA
      READ(5,61) NFILT, NEG, LGRID, STOPF, PASSF, PRF, RATIO, WEIGHT
```

TABLE C-1. (Continued)

```
   61  FORMAT(I3,I2,I5,F5.3,F10.5,2F5.1,10X,F5.2)
       EDGE(2)= STOPF / PRF
       EDGE(3)= PASSF / PRF
       NFCNS = NFILT / 2
       NODD = NFILT - 2*NFCNS
       IF(NODD.EQ.1) NFCNS= NFCNS+1
C** PROGRAM DESIGN MAX 150 TAPS / LGRID = 16.
       IF(LGRID.LE.0) LGRID = 1200 / NFCNS
       ITEST = LGRID * NFCNS
       IF(ITEST.LF.1200) GO TO 66
       WRITE(6,65) NFILT, LGRID, ITEST
   65  FORMAT(50H1---- INPUT DATA EXCEEDS ALLOWABLE DIMENSION ----.//
      *      * NFILT = *,I3,* LGRID = *,I3,* ITEST = *,I5)
       GO TO 500
C
C SET UP DENSE GRID WITH NO. GRID PTS..LE. LGRID * (NFILT + 1) / 2
C      FIND THE DESIRED MAGNITUDE (DES(J)) AND WEIGHT (WT(J)) ON GRID.
    66 GRID(1)=EDGE(1)
       DELF = 0.5 / FLOAT(LGRID * NFCNS)
       J=1
       FUP=EDGE(2)
       IF(NEG.EQ.0) GO TO 145
       GRID(1) = DELF
   145 TEMP=GRID(J)
       DES(J)= 0.00
       WT(J)= WEIGHT * (1.00-(1.00-RATIO)*TEMP/EDGE(2))
       J=J+1
       GRID(J)=TEMP+DELF
       IF(GRID(J).GT.FUP) GO TO 150
       GO TO 145
   150 GRID(J-1)=FUP
       IGRID= J
       WT(J-1)= RATIO * WEIGHT
       GRID(J)= EDGE(3)
```

TABLE C-1. (Continued)

```
      FUP= 0.50
  155 TEMP= GRID(J)
      DES(J)= 1.00
      WT(J)= 1.00
      J= J + 1
      GRID(J)= TEMP + DELF
      IF (GRID(J).GT.0.50) GO TO 160
      GO TO 155
  160 GRID(J-1)= 0.50
      NGRID= J-1
C     S E T   U P   A P P R O X I M A T I O N   P R O B L E M .
      IF (NEG.EQ.1) GO TO 175
      IF(NODD.EQ.1) GO TO 180
C  PLOT WT * COS(PI*F) VS. FREQUENCY (EVEN,POS.)
      IF(GRID(NGRID).GT.(0.5-DELF)) NGRID=NGRID-1
      DO 170 J=1,NGRID
      CHANGE= COS(PI*GRID(J))
      DES(J)=DES(J)/CHANGE
  170 WT(J)=WT(J)*CHANGE
      WRITE (6,171)
  171 FORMAT (12H1X = GRID(J),5X,20HY = WT(J)*COS(PI*X) )
      GO TO 185
C  PLOT WT * SIN(PI*F) VS. FREQUENCY (EVEN,NEG.)
 175  DO 176 J=1,NGRID
      CHANGE = SIN(PI*GRID(J))
      DES(J)= DES(J) / CHANGE
  176 WT(J)= WT(J) * CHANGE
      WRITE (6,177)
  177 FORMAT (13H1 X = GRID(J),5X,20H Y = WT(J)*SIN(PI*X) )
      GO TO 185
C  PLOT WT VS. FREQUENCY (ODD,POS.)
  180 WRITE (6,181)
  181 FORMAT (13H1 X = GRID(J),5X,10H Y = WT(J) )
 185  CONTINUE
```

TABLE C-1. (Continued)

```
C
C INITIAL GUESS FOR EXTREMAL FREQUENCIES IS EQUALLY SPACED ON GRID
      TEMP=FLOAT(NGRID-1)/FLOAT(NFCNS)
      DO 210 J=1,NFCNS
  210 IEXT(J)=(J-1)*TEMP+1
      IEXT(NFCNS+1)=NGRID
      NM1=NFCNS-1
      NZ=NFCNS+1
      CALL REMEZ(EDGE,2)
C     C A L C U L A T E   I M P U L S E   R E S P O N S E .
      IF (NEG.EQ.1) GO TO 320
      IF(NODD.EQ.0) GO TO 310

      DO 305 J=1,NM1
  305 H(J)=0.5*ALPHA(NZ-J)
      H(NFCNS)=ALPHA(1)
      GO TO 350
  310 H(1)=0.25*ALPHA(NFCNS)
      DO 315 J=2,NM1
  315 H(J)=0.25*(ALPHA(NZ-J)+ALPHA(NFCNS+2-J))
      H(NFCNS)=0.5*ALPHA(1)+0.25*ALPHA(2)
      GO TO 350
  320 H(1)= 0.25 * ALPHA(NFCNS)
      H(NFCNS)= 0.50*ALPHA(1) - 0.25*ALPHA(2)
      DO 325 J=2,NM1
  325 H(J)= 0.25 * (ALPHA(NZ-J)-ALPHA(NFCNS+2-J))
C
C     P R O G R A M   O U T P U T   S E C T I O N .
  350 WRITE(6,360)
  360 FORMAT(1H0, 70(1H*),//,10X,*FINITE IMPULSE RESPONSE (FIR)*,
     *    * LINEAR PHASE DIGITAL FILTER DESIGN * / 10X,
     *    * FOR REMOVING GROUND CLUTTER IN MTI RADAR SIGNAL PROCESSOR*//)
      IF (NEG.EQ.1) GO TO 365
      WRITE (6,361) LGRID
```

TABLE C-1. (Continued)

```
  361 FORMAT ( 2X,*IMPULSE RESPONSE WITH POSITIVE SYMMETRY (LGRID = *
     1       ,I3,*)*/ )
      GO TO 369
  365 WRITE (6,366) LGRID
  366 FORMAT ( 2X,*IMPULSE RESPONSE WITH NEGATIVE SYMMETRY (LGRID = *
     1       ,I3,*)*/ )
C  CALCULATE NOISE POWER GAIN (PGN) IN DB
 369  SUM= 0.00
      DO 381 K=1,NFCNS
      Z(K)= H(K)
      L= NFILT + 1 - K
      Z(L)= Z(K)
      IF (NEG.EQ.1) Z(L)= -Z(L)
 381  SUM= SUM + 2.00 * Z(K)**2
      IF (NODD.EQ.1) SUM = SUM - Z(NFCNS)**2
      PGN= 10.00 * ALOG10(SUM)
      SRTSUM = SQRT(SUM)
      WRITE (6,383) PGN , (Z(J),J=1,NFILT)
  383 FORMAT(1H0,  * ORIGINAL TAP GAINS WITH NOISE POWER GAIN = *,F7.3,
     1 * DB * //(2X,10F10.5/))
C  NORMALIZE H(K) W.R.T. PGN AND COMPUTE NEW PGN = 0 DB
      SUM = 0.
      DO 384 K = 1,NFILT
      Z(K) = Z(K)/ SRTSUM
  384 SUM = SUM + Z(K)**2
      PGN = 10.* ALOG10(SUM)
      WRITE (6,385) PGN , (Z(J),J=1,NFILT)
  385 FORMAT(1H0,* NORMALIZED TAP GAINS WITH NOISE POWER GAIN = *,F7.3,
     1 * DB * //(2X,10F10.5/))
      WRITE(6,390)
  390 FORMAT(1H0,20X,35(1H+),//31H  BAND  LOWER EDGE  UPPER EDGE  ,5X
     *       ,*WEIGHT*,5X,*RATIO*,5X,*RIPPLE*,5X,*RIPPLE(DB)*/2X,76(1H-))
      DEVIAT = DEV / WEIGHT
      HPRF= PRF / 2.00
      DBSTOP = -20.* ALOG10(DEVIAT/(1.+DEV))
      DBPASS = 20.* ALOG10( (1.+ DEV)/(1.- DEV) )
```

TABLE C-1. (Concluded)

```
      WRITE(6,410) STOPF, WEIGHT, RATIO, DEVIAT, DBSTOP
  410 FORMAT(2X,*STOP        0.00*,2X,F10.2,6X,F6.2,6X,F4.2,4X,F7.5,5X,
     *      F10.3 / )
      WRITE (6,420) PASSF, HPRF, DEV, DBPASS
  420 FORMAT (2X,*PASS*,2(5X,F7.2),8X,*1.00*,14X,F7.5,5X,F10.3)
      DO 450 J=1,NZ
 450  EXTF(J)= GRID(IEXT(J))*PRF
      WRITE(6,455) (EXTF(J),J=1,NZ)
  455 FORMAT(2H   ,76(1H-)//*  EXTREMAL FREQUENCIES(SCALED WRT PRF) *
     *       // (8F10.3) )
C
C     P L O T   F R E Q U E N C Y   R E S P O N S E   I N   D B .
C     THIS CODE GENERATES FILTER COEFFICIENTS INTO THE Z ARRAY.
C     THE USER IS RESPONSIBLE FOR PROVIDING VALUES FOR THE FOLLOWING
C     VARIABLE PARAMETERS. NFILT, NP, SFREQ, FFREQ, PRF,  AND EPLUN.
      SFREQ = 0.
      FFREQ = 750.
      NP = 151
      WRITE (6,470) NFILT, PRF, PGN
  470 FORMAT (/2X,*FREQUENCY RESPONSE FOR*,I3,* TAP MTI FILTER, PRF = *,
     1        F6.0,* HZ, NOISE POWER GAIN = *,F7.3,* DB *,//,
     2 * X = FREQ. (HZ)   Y = 20 LOG10(MAG) - PGN*)
      CALL AMPSQDB(Z,NFILT,NP,SFREQ,FFREQ,PRF,1.0E-7,PGN,HDB,FREQ)
      CALL GRAPH1(FREQ,HDB,NP,1)
  500 CONTINUE
      STOP
      END
```

Appendix D.

MTIDSN – PROGRAM TO SELECT WEIGHT AND PASSF

The program MTIDSN is capable of providing values for WEIGHT and PASSF as required to complete the design of an MTI filter. It uses the same design specification card as program MTI plus values for ASB and RPB with blanks left for the unknown parameters. This program computes the value of WEIGHT = δ/Δ, by solving Equations (3) and (4) for δ and Δ, respectively, and then obtains an initial estimate for PASSF. In theory, any of the filter parameters could be treated as the unknown. Details are found in the paper by Rabiner [10] for LPF design and the necessary modifications for HPF design are quite straightforward. After an initial PASSF estimate is obtained, the design parameters are passed to subroutine HPF which computes the new value for stopband ripple (DELTA2). This value is compared with the specified value, determined by ASB and RPB specifications. Depending on the comparison, the value of PASSF is increased if DELTA2 (Δ) is too large or decreased if DELTA2 is too small. If DELTA2 is within 1% of the specified value, the search ends. A listing for program MTIDSN and subroutine HPF, which is a streamlined version of Program MTI without all the output data statements, are found in Tables D-1 and D-2, respectively.

TABLE D-1. PROGRAM MTIDSN LISTING

```
      PROGRAM MTIDSN(INPUT,OUTPUT,TAPE5,TAPE6)
C***********************************************************************
C** MTIDSN DETERMINES THE APPROPRIATE VALUE OF THE UNKNOWN DESIGN PAR.**
C** (NFILT,STOPF,PASSF,ASTOP,RPASS).  THE ALGORITHM IS DESCRIBED BY   **
C** RABINER, IEEE TR. AUDIO, OCT 73, AND INTERACTS WITH THE SUBROUTINE**
C** HPF, AN MTI FILTER DESIGN PROGRAM BASED ON MCCLELLANS ALGORITHM, **
C** IEEE TR. AUDIO, DEC 73.  THE PROGRAM IS CURRENTLY ABLE TO SPECIFY **
C** -- NFILT, PASSF                                                  **
C** THE MTI DESIGN PARAMETERS INCLUDE                                **
C**    1. NFILT - NUMBER OF FILTER WEIGHTS (.LE. 150)                **
C**    2. NEG - TAP SYMMETRY ABOUT NTAPS/2. POS.(=0), NEG.(=1)       **
C**    3. LGRID - GRID DENSITY. (LGRID*NTAPS/2 .LE. 1200)            **
C**    4. STOPF - UPPER EDGE OF STOPBAND (HZ).                       **
C**    5. PASSF - LOWER EDGE OF PASSBAND (HZ).                       **
C**    6. PRF - PULSE REPETITION FREQUENCY (HZ)                      **
C**    7. RATIO - WTX(STOPF) / WTX(0.)   (.LE. 1)                    **
C**    8. ASB   - STOPBAND ATTENUATION (DB)                          **
C**    9. RPB   - PASSBAND RIPPLE (DB)                               **
C***********************************************************************
C** MODIFICATION WHICH FINDS PASSBAND EDGE (PASSF) GIVEN NFILT
      READ(5,5) NRUNS
    5 FORMAT(I2)
      PI = 3.141592654
      DO 100 I=1,NRUNS
      READ(5,10) NFILT,NEG,LGRID,STOPF,PASSF,PRF,RATIO,ASB,RPB
   10 FORMAT(I3,I2,I5,F5.3,F10.5,4F5.1)
      WRITE(6,20) NFILT,ASB,RPB,STOPF,PASSF,PRF
   20 FORMAT(1H0,20X,* DESIGN SPECIFICATIONS FOR MTI FILTER * //
     *     * NUMBER TAPS = *,  I3 , * , STOPBAND ATTENUATION = *, F4.0,
     *      * DB , PASSBAND RIPPLE = *, F4.1, * DB * //
     *    * STOP FREQ. = *,F8.4,* HZ , PASS FREQ. = *,F8.4,* HZ , PRF = *
     *      ,F6.1, * HZ * // )
      FCU = STOPF / PRF
      CRP = 10.**(RPB / 20.)
```

TABLE D-1. (Continued)

```
      D1 = (CRP - 1.)/(CRP + 1.)
      D2 = 10.**(-ASB / 20.)*(1.+ D1)
      WTX = D1 / D2
      IF(FCU .GE. 0.04) GO TO 35
C  CHEBYSHEV LOWFR BOUND ESTIMATE FOR FCU .LT. 0.04
      X = (1. + D1) / D2
      Y = (1. - D1) / D2
      COSHIX = ALOG( X + SQRT(X**2-1.) )
      COSHIY = ALOG( Y + SQRT(Y**2-1.) )
      DELF = ( COSHIX - SQRT( COSHIX**2 - COSHIY**2 ) )/(PI *(NFILT-1))
      GO TO 45
C  HERRMANN ESTIMATE (BSTJ) FOR FCU .GE. 0.04
   35 D1L = ALOG10(D1)
      D2L = ALOG10(D2)
      FK = 11.01217 + 0.51244 * ALOG10(WTX)
      DINF = (0.005309 * D1L**2 + 0.07114 * D1L - 0.4761)*D2L
     *        - 0.00266 * D1L**2 - 0.5941 * D1L - 0.4278
      DELF = (NFILT-1)*(SQRT( 1.+ 4.*FK * DINF /(NFILT-1)**2)- 1.)
     *   / (2.* FK)
   45 FUP = FCU + DELF
      IF(FUP .GE. 0.4) FUP = 0.4
      FST = DELF / 10.
      ITER = 0
      JD = 0
      DEL = 0.01 * D2
      WRITE(6,30) LGRID, RATIO, NEG, ITER, NFILT, FCU, FUP, D1, D2, WTX
   30 FORMAT(1H0,10X,*DESIGN VALUES GIVEN ( LGRID = *,I3,* , RATIO = *,
     *       F5.3,* , AND NEG = *,I1, * ) * //
     *       * ITER. NTAPS  STOP/PRF  PASS/PRF  DELTA1  DELTA2  WEIGHT *
     *    // 3X,I2,4X,I3,4X,F5.3,4X,F6.4,3X,F6.4,1X,F8.6,2X,F5.1 )
   49 IF(FUP .LE. 0.45) GO TO 50
      WRITE(6,40) FUP
   40 FORMAT(1H0,*+++UNSUCCESSFUL DESIGN BECAUSE PASSF/PRF = *,F6.4,
     *    * , WHICH IS GREATER THAN 0.4 +++* )
      GO TO 100
```

TABLE D-1. (Concluded)

```
50  CALL HPF(NFILT,FCU,FUP,WTX,RATIO,NEG,LGRID,ESDEL1,ESDEL2)
    ITER = ITER + 1
    WEIGHT = ESDEL1 / ESDEL2
    WRITE(6,60) ITER,  FUP,  ESDEL1, ESDEL2
60  FORMAT( 1H0,2X,I2,20X,F6.4,3X,F6.4,1X,F8.6)
    ERROR = D2 - ESDEL2
    TEST = ABS(ERROR)
    IF(TEST .LE. DEL) GO TO 55
    IF(ERROR .LE. 0) GO TO 65
    IF(JD .EQ. 1) FST = FST / 3.
    FUP = FUP - FST
    JD = - 1
    GO TO 75
55  JD = 2
    GO TO 75
65  IF(JD .EQ. -1) FST = FST / 3.
    FUP = FUP + FST
    JD = 1
75  IF(JD.NE. 2) GO TO 49
    PASSF = FUP * PRF
    WRITE(6,80) ITER, PASSF, WEIGHT
80  FORMAT(1H0,* +++ SUCCESSFUL DESIGN AFTER *,I2, * ITERATIONS +++ *
   *       // 10X,* PASSF = *,F8.3, * HZ  AND  WTX = *,F6.2 // )
100 CONTINUE
    STOP
    END
```

TABLE D-2. SUBROUTINE HPF LISTING

```
      SUBROUTINE HPF(NFILT,STOPF,PASSF,WEIGHT,RATIO,NEG,LGRID,ESDEL1,
     *               ESDEL2)
      COMMON PI2,AD,DEV,X,Y,GRID,DES,WT,ALPHA,IEXT,NFCNS,NGRID
      DIMENSION IEXT(79),AD(79),ALPHA(79),X(79),Y(79),EDGE(4),
     *          DES(1200),GRID(1200),WT(1200)
      PI = 3.14159265359
      PI2 = 6.28318530718
      EDGE(1) = 0.
      EDGE(2) = STOPF
      EDGE(3) = PASSF
      EDGE(4) = 0.5
      NFCNS = NFILT / 2
      NODD = NFILT - 2*NFCNS
      IF(NODD.EQ.1) NFCNS= NFCNS+1
C** PROGRAM DESIGN MAX 150 TAPS / LGRID = 16.
      IF(LGRID.LE.0) LGRID = 16
      ITEST = LGRID * NFCNS
      IF(ITEST.LE.1200) GO TO 140
      NFILT = 1200
      GO TO 200
C**   FIND THE DESIRED MAGNITUDE (DES(J)) AND WEIGHT (WT(J)) ON GRID.
  140 GRID(1) = EDGE(1)
      DELF = 0.5 / FLOAT(LGRID * NFCNS)
      J=1
      FUP=EDGE(2)
      IF(NEG.EQ.0) GO TO 145
      GRID(1) = DELF
  145 TEMP=GRID(J)
      DES(J)= 0.
      WT(J)= WEIGHT * (1.00-(1.00-RATIO)*TEMP/EDGE(2))
      J=J+1
      GRID(J)=TEMP+DELF
      IF(GRID(J).GT.FUP) GO TO 150
      GO TO 145
```

TABLE D-2. (Concluded)

```
  150 GRID(J-1)=FUP
      WT(J-1)= RATIO * WEIGHT
      GRID(J)= EDGE(3)
      FUP= 0.50
  155 TEMP= GRID(J)
      DES(J)= 1.00
      WT(J)= 1.
      J= J + 1
      GRID(J)= TEMP + DELF
      IF (GRID(J).GT.0.50) GO TO 160
      GO TO 155
  160 GRID(J-1)= 0.50
      NGRID= J-1
C** SET UP APPROXIMATION PROBLEM.  WEIGHT BY SIN(PI*GRID(J)) IF NFILT EVEN
      IF(NODD.EQ.1) GO TO 185
      DO 175 J=1,NGRID
      CHANGE = SIN(PI*GRID(J))
      DES(J)= DES(J) / CHANGE
  175 WT(J)= WT(J) * CHANGE
C INITIAL GUESS FOR EXTREMAL FREQUENCIES IS EQUALLY SPACED ON GRID
  185 TEMP=FLOAT(NGRID-1)/FLOAT(NFCNS)
      DO 190 J=1,NFCNS
  190 IEXT(J)=(J-1)*TEMP+1
      IEXT(NFCNS+1)=NGRID
      CALL REMEZ(EDGE,2)
      ESDEL1 = DEV
  200 ESDEL2 = DEV / WEIGHT
      RETURN
      END
```

Appendix E.

ESTTAP – PROGRAM TO ESTIMATE NFILT

Program ESTTAP is used to predict the number of weights (NFILT) required to achieve a particular normalized passband (FUP = PASSF/PRF) for ASB, RPB, and normalized stopband (FCU = STOPF/PRF). User inputs are contained on one card which includes values for FCU, ASB, and RPB. Output includes eight pairs of NFILT, FUP values plus values of the passband ripple (D1), stopband ripple (D2), and their ratio WTX (= D1/D2). The increments for FUP are FCU/4 if FCU $\geq$ 0.04 and FCU otherwise. The value of NFILT is a close approximation to the actual value if FCU $\geq$ 0.04 and D1 $\leq$ 0.1. The algorithm for this situation is based on an empirical equation described by Herrmann, et. al., [11] and is usually accurate within $\pm$2 weights. If D1 > 0.1, the calculations are performed, but an accuracy disclaimer is printed. If FCU < 0.04, the resulting NFILT is an estimate based on the Chebyshev polynomial which describes the approximation error. The estimate is a lower bound for NFILT odd, but is an upper bound for even values. A listing for the program is given in Table E-1. The program is fast and multiple designs are handled by preceding the set of design cards with a card indicating the number of designs (NRUNS).

TABLE E-1. PROGRAM ESTTAP LISTING

```
      PROGRAM ESTTAP(INPUT,OUTPUT,TAPE5,TAPE6)
C***********************************************************************
C** ESTIAP IS DESIGNED TO ESTIMATE THE NUMBER OF TAPS REQUIRED TO     **
C** DESIGN AN MTI FILTER GIVEN STOPF/PRF (FCU) AND THE DB RESPONSE IN **
C** STOP(ASB) AND PASS(RPB) BANDS.  ALGORITHM DESCRIBED BY HERRMANN,  **
C** BSTJ, JULY 73.  TWO ESTIMATES ARE POSSIBLE.                        **
C**   1. FCU.LT. 0.04 - CHEBYSHEV EQN. - LOWER BOUND FOR N ODD, BUT    **
C**      UPPER BOUND FOR N EVEN.                                       **
C**   2. FCU.GE. 0.04 - EMPERICAL FIT - ESTIMATE NOT GOOD FOR LARGE    **
C**      PASSBAND RIPPLE (.GT. 1 DB)                                   **
C** PROGRAM PROVIDES ESTIMATES FOR 8 TRANSITION BANDWIDTHS (FINC)     **
C** STARTING WITH FCU/4 IF FCU.GE. 0.04 OR FCU IF FCU.LT. 0.04 AND     **
C** INCREMENTS ACCORDINGLY.                                            **
C***********************************************************************
      READ(5,5) NRUNS
    5 FORMAT(I2)
      DO 100 J = 1,NRUNS
      READ(5,10) FCU, ASB, RPB
   10 FORMAT(3F5.3)
      WRITE(6,20) FCU, ASB, RPB
   20 FORMAT(1H0,20X,* DESIGN SPECIFICATIONS FOR MTI FILTER * //
     *      * CUTOFF/PRF = *, F5.3, * , STOPBAND ATTENUATION = *, F4.0,
     *      * DB , PASSBAND RIPPLE = *, F5.3, * DB * )
      CRP = 10.**(RPB / 20.)
      D1 = (CRP - 1.)/(CRP + 1.)
      D2 = 10.**(-ASB / 20.) * (1.+ D1)
      WTX = D1 / D2
      WRITE(6,30) D1, D2, WTX
   30 FORMAT(1H0, * DELTA-1 = *, F7.5,9X,*DELTA-2 = *, F10.8,15X,
     *       * WEIGHT RATIO = *, F6.1 / )
      IF(D1.LE.0.1) GO TO 45
      WRITE(6,40)
   40 FORMAT(* ----- PASSBAND RIPPLE EXCEEDS 0.1 ----- *,10X, * CONSULT
     *BSTJ, VOL.52, NO.6, P.791 * )
```

TABLE E-1. (Concluded)

```
   45 D1L = ALOG10(D1)
      D2L = ALOG10(D2)
      FK = 11.01217 + 0.51244 * ALOG10(WTX)
      DINF = (0.005309 * D1L**2 + 0.07114 * D1L - 0.4761)*D2L
     *       - 0.00266 * D1L**2 - 0.5941 * D1L - 0.4278
      TAPEST = 0.
      CN = 0.
      FUP = FCU
      DELF = 0.
      FINC = 0.25 * FCU
      IF(FCU .LT. 0.04) FINC = FCU
      WRITE(6,50) FCU
   50 FORMAT(1H0, * EFFECT OF VARYING UPPER BANDEDGE ON TAP ESTIMATE WIT
     *H CUTOFF FREQ./PRF = *,F5.3//20X,* ITER.  FUP/PRF  TAP(EST.) TAP(L
     *.B.) * / )
C** INVESTIGATE EIGHT UPPER EDGES FOR A GIVEN CUTOFF FREQUENCY
      DO 65  I = 1,8
      DELF = DELF + FINC
      FUP = FCU + DELF
      IF(FUP.GT.0.4) GO TO 100
      IF(FCU.GE.0.04) GO TO 55
C** CHEBYSHEV APPROX. (LOWER BOUND ON NTAPS FOR FCU .LT. 0.04 * PRF)
      X = (1. + D1) / D2
      Y = 1. / COS(3.141592654 * FUP)
      CN = 1. + ALOG(X + SQRT(X**2 - 1.) )/ ALOG(Y + SQRT(Y**2 - 1.) )
      GO TO 65
   55 TAPEST = DINF / DELF - FK * DELF + 1.
   65 WRITE(6,60) I, FUP, TAPEST, CN
   60 FORMAT(23X,I1,3X,F7.5,4X,F6.1,5X,F5.1)
      WRITE(6,70)
   70 FORMAT(20X,36(1H+) // )
  100 CONTINUE
      STOP
      END
```

Appendix F.

SIGNAL–TO–NOISE CONSIDERATIONS FOR EAR

The signal-to-noise ratio (SNR) improvement required in the MTI signal processor is a function of the P_d and the input signal-to-noise (ISN) ratio. Typically, for a specified $\dot{P}_f$ of 10^{-6}, a $P_d = 0.5$ can be achieved for a fluctuating target with an output signal-to-noise (OSN) ratio of 13 dB [12]. It is assumed that the noise is uniformly distributed throughout the IF bandwidth. The value of ISN is a function of range (R) in meters and is determined by the range equation

$$ISN = 10 \log \left[\frac{P\ G^2\ \lambda^2\ C}{(4\pi)^3\ R^4\ kT\ B\ \overline{NF}\ L} \right] , \qquad \text{(F-1)}$$

where the symbols are defined in Table F-1 and typical values assigned for the EAR system. Using the gain-loss data in Table F-1, it is possible to express ISN as a function of R alone,

$$ISN = 163 - 40 \log(R) \text{ dB} \quad . \qquad \text{(F-2)}$$

The values of ISN for some typical ranges are found in Table F-2. To illustrate the effect of range on signal processor design, consider the case where all targets out to a range of 10 km must be detected in accordance with the aforementioned error probability. Consequently, with ISN = 3 dB, it follows that the processor must provide 10 dB of gain. Such gain can be obtained by summing 16 outputs from a fixed window TPC. It follows from Table F-2 that the integrator provides 12-dB gain and that the detection specifications are met if HDB(f) > -2 dB, i.e., over some 51% of the PRF interval. Conversely, for a target at 5 km, the signal processor can exhibit a loss of 2 dB, i.e., HDB(f) = -14 dB for the TPC, and still achieve the necessary P_d. This corresponds to a useable bandwidth which covers 77% of the PRF interval. Alternatively, a 48-tap Chebyshev MTI filter with a passband ripple RPB = 4 dB could be employed without an integrator and the usable passband would cover more than 90% of the PRF interval.

TABLE F-1. GLOSSARY FOR SYMBOLS USED IN RANGE EQUATION

Symbol	Definition	Value	dB
B	IF bandwidth (Hz)	$(0.2\ \mu sec)^{-1}$	67
C	Target cross section (m^2)	1.5	2
G^2	Two-way antenna gain	-	50
kT	Boltzmann's constant × 290°K	4×10^{-21}	-204
L	Estimated system losses	-	12
$\overline{NF}$	Operating noise factor	-	6
P	Transmitter power (W)	10^5	50
R	Target range (m)	Variable	40 log(R)
λ	Radar wavelength (m)	0.0545	-25
$(4\pi)^3$	Constant	$(12.56)^3$	33

TABLE F-2. INPUT SIGNAL-TO-NOISE RATIO AS A FUNCTION OF RANGE

R (km)	ISN (dB)
5.0	15
7.5	8
10.0	3
12.5	-1
15.0	-4
17.5	-7
20.0	-9

REFERENCES

1. Barton, D. K., Radar System Analysis, Prentice-Hall, Englewood Cliffs, New Jersey, 1964, p. 100.

2. Lancaster, J. F., MICOM Experimental Array Radar Digital Signal Processor, Report No. RE-TR-73-14, US Army Missile Command, Redstone Arsenal, Alabama, February 1973, p. 7.

3. Barton, D. K., op. cit., pp. 211, 219.

4. Capon, J., "Optimum Weighting Functions for the Detection of Sampled Signals in Noise," IEEE Trans. Information Theory, April 1964, pp. 152-159.

5. Fletcher, R. H., Jr., and Burlage, D. W., Improved Moving-Target-Indicator Filtering for Phased-Array Radars, Report No. RE-TR-73-17, US Army Missile Command, Redstone Arsenal, Alabama, March 1973.

6. Fletcher, R. H., Jr., Recursive Digital Filter Design and Analysis with Applications to Radar Processing, Report No. RE-TR-73-6, US Army Missile Command, Redstone Arsenal, Alabama, April 1973.

7. McClellan, J. H., Parks, T. W., and Rabiner, L. R., "A Computer Program for Designing Optimum FIR Linear Phase Digital Filters," IEEE Trans. on Audio and Electroacoustics, December 1973, pp. 506-526.

8. Ewell, G. W., III, "Design of Digital Moving Target Indication Radar Processors," Ph.D. Thesis, Georgia Institute of Technology, April 1974.

9. Skolnik, M. I., Radar Handbook, McGraw-Hill, New York, 1970, pp. 17-38 to 17-46.

10. Rabiner, L. R., "Approximate Design Relationships for Low-Pass FIR Digital Filters," IEEE Trans. Audio and Electroacoustics, October 1973, pp. 456-460.

11. Herrman, O., Rabiner, L. R., and Chan, D. S. K., "Practical Design Rules for Optimum Finite Impulse Response Low-Pass Digital Filters," Bell Sys. Technical Journal, July 1973, pp. 769-799.

12. Barton, D. K., op. cit., p. 23.

GLOSSARY

Name	Definition
ASB	Specified stopband attenuation (dB)
BW	Useable bandwidth of MTI filter (Hz)
CHEB	Chebyshev filter
COV	Covariance filter
EAR	Experimental array radar
H(f)	Frequency response of MTI filter
HDB(f)	Value of H(f) (dB)
HPB_m	Minimum passband value of HDB(f)
HPF	High-pass filter
I	Improvement factor (dB)
IG	Integration gain (dB)
ISN	Signal-to-thermal-noise ratio (dB) at signal processor input
MTI	Moving target indicator
N	Number of taps (weights) used in MTI filter
NFILT	Value of N used in Chebyshev design program
OSN	Signal-to-thermal-noise ratio (dB) at signal processor output
P_d	Probability of detection for given OSN and P_f
P_f	Probability of false-alarm for given threshold
P(f)	Normalized AC clutter power-density spectrum
PASSF	Lower edge of passband (Hz) as used in Chebyshev design program
PRF	Pulse repetition frequency (Hz)
R	Number of MTI outputs summed by integration
RATIO	Ratio of Δ to H(f) at STOPF
RPB	Specified passband ripple (dB)
SCR	Signal-to-clutter ratio (dB)
SNR	MTI processor gain (dB) OSN - ISN
STOPF	Upper edge of stopband in Hz as used in Chebyshev design program
T	Pulse repetition interval (sec)

Name	Definition
TPC	Three-pulse canceller
WEIGHT	δ/Δ
f_d	Doppler frequency of the target (Hz)
f_m	Minimum frequency (Hz) for which HDB(f) = SNR - IG
h_i	Weight (multiplicand) of the i^{th} tap (multiplier) in a MTI filter
m^2	Ratio of DC-to-AC clutter power
δ	Maximum passband ripple (error)
Δ	Nominal stopband ripple (error)
σ	Standard deviation of Gaussian clutter spectrum (Hz)
σ_d	Value of σ used in MTI design considerations